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Analysis of the Impact of Percentage Variation of the Intrinsic Growth Rate Parameter Value in the interaction between $PM_{2.5}$ and Relative Humidity using Computational Method

[1]U.F. Onyeananam; & [2]R.E. Akpodee

[1]Institute of Geoscience and Environmental Management, Faculty of Science, Rivers State University. [2]Department of Mathematics, Faculty of Science, Rivers State University.

Corresponding Author: udonyeananam@hotmail.com
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Abstract

In the study of Analysis of the Impact of Percentage Variation of the Intrinsic Growth Rate Parameter Value in the interaction between $PM_{2.5}$ and Relative Humidity using Computational Method, sensitivity analysis was carried out by decreasing and increasing the intrinsic growth rate parameter of pollutant level of particle size 2.5 in terms of percentage variation, we have adopt the mathematical modelling approach of numerical simulation using Matlab ODE45 numerical scheme for a time interval of 0(30)360 months. We observed that due to a 50% variation of the growth rate of the pollutant level 2.5 at an interval of 30days for a total time period of 366 days, we observed that the modified value of the $PM_{2.5}$ experienced a drastic depletion down the trend from an initial value of 19.40 and decreased to a converging saturated value of 10.9626 in amount over an optimal time of 360days with the relative humidity having a slight gain from an initial value of 62.0 which increases to a converging saturated value of 72.9722 in amount over an optimal time of 360days when compared to the fixed coordinates and the quantification which measures this severe depletion and slight gain were also computed as well. Furthermore, due to a 150% variation of the growth rate of the pollutant level 2.5 at an interval of 30days for a total time period of 366 days, we observed that the modified value of the $PM_{2.5}$ experienced a strong gained down the trend from an initial value of 19.40 and increases to a converging saturated value of 33.6199 in amount over an optimal time of 360days with the relative humidity having a slight depletion from an initial value of 62.0 which decreases to a converging saturated value of 68.9830 in amount over an optimal time of 360days when compared to the fixed coordinates and the quantification which measures this severe depletion and slight gain were also computed as well. The full results and discussion of this novel contributions to knowledge are fully presented in this work.

Keywords: Numerical Scheme, Vulnerability, Environmental Pollution, Mathematical Modelling, Dynamical System, Meteorological Variable.

Introduction

Despites the several contributions of mathematical modeling to understand the sophisticated dimension of chemo dynamics phenomena, it is rare to find research contribution that with the numerical simulation aspect of chemo dynamics of which the interaction between pollutant level 1 and the relative humidity is a special scenario. It is against this background that we have proposed in this pioneering study to investigate the quantitative characterization of this type of interaction by using a computational approach that is driven by the idea of artificial intelligence. To the best of our knowledge, the notion of artificial intelligence can be briefly conceptualized to be a combination of general knowledge and a detailed knowledge. The inclusion of a detailed knowledge is imperative in our present mathematical analysis.

Modelling the interaction between pollutant level ($PM_{2.5}$) and Relative Humidity R_H in the Niger-Delta region of Nigeria will greatly help in prescribing safety limits for community and occupational exposure in this region of our country. The formulation of guidelines aimed at the protection of ecosystems from environmental hazard such as air-borne diseases, Health hazards and health diseases. High humidity effect on the ecosystem could also be more effective since ecosystem functioning data pertaining to this subject are limited inferences have to be drawn from simulated experimental studies and case reports. The basis of the present overview is the information available from literature on related temperature and humidity effects on the environment and ecosystem functioning. The effects of relative humidity and pollutant level 1 and 2.5 on air quality and impact on the ecosystem in the Niger Delta region of Nigeria will be well studied.

Air pollutants whose concentrations and impacts are known to be affected by heatwaves include ozone (O_3), particulate matter (PM) and nitrogen dioxide (NO_2). These pollutants, when emitted into the atmosphere from a variety of natural and anthropogenic sources, are a major threat to human health (World Health Organization, 2017). The concentration of these pollutants in ambient air depends on the level of emission and the ability of the atmosphere to absorb or disperse these pollutants (World Health Organization, 2017). Since meteorological variables such as relative humidity and concentrations of air pollutants vary on a daily basis, it is important to consider their relationship in the planetary boundary layer, since the atmosphere is the medium in which air pollutants are transported away from the source (Ekaba *et al.,*2022). Gee and Sollans, 1998, Gobo et al., 2012 indicated that during photochemical pollution episodes, air pollutants (O_3, PM_{10}, and NO_2) are the result of a mixture of various meteorological effects and chemical reactions. These pollutants are of the greatest health concern, as their emissions may be exacerbated during heatwaves (Fenger,1999), Gobo *et al*. (2010)). As a result, sensitive individuals may not only be stressed by high temperatures, but may be more subject to mortality due to air pollution during heatwaves. Unusually hot weather during summer led to elevated levels of air pollutants during the heatwaves that occurred in Athens, Greece, between June and July 2007 (Paul *et al*., 2004). He found a significant correlation between heatwaves and average hourly concentrations of O_3, NO_2 and SO_2 in Athens.

Similarly, the combination of elevated air pollution levels and high temperatures was implicated in the increase in urban heat islands (UHIs) and air pollution in London, England (McMichael *et al*., 2003; Rooney *et al*., 1998). Heatwaves are particularly intense in urban areas, where surface characteristics alter the temperature differences between urban and rural areas. The differences are generated by low levels of vegetation in cities, and the production of anthropogenic heat and air flow caused by extreme air temperatures in cities (UHIs) increase heat- and air pollution-related mortality and raise the energy demands for cooling buildings, which in turn leads to a further increase in air pollutants and greenhouse gas emissions. Despite this, several studies have shown that sustainable urban planning, and smart city design including green roofs and cool pavements could significantly reduce both UHIs and air pollution; especially O_3, NO_2, andPM_{10} (Bibri & Krogstie, 2017; McDonald *et al*., 2007; Silva *et al*., 2018; Yang *et al*., 2008).

Mathematical Model Formulation

Following Lotka (1956), our model formulation for the interaction between relative humidity and particulate matter $PM_{2.5}$ is based on a system of two non-linear first-order ordinary differential equations stated as follows:

$$\frac{d\mathrm{PM}_{2.5}}{dt} = \mathrm{PM}_{2.5}[\alpha_1 - \beta_1 \mathrm{PM}_{2.5} - \gamma_1 R_h]\,, \quad \mathrm{PM}_{2.5}(0) = \mathrm{PM}_0 > 0$$
$$\frac{dR_h}{dt} = R_h[\alpha_2 - \beta_2 R_h - \gamma_2 \mathrm{PM}_{2.5}],\; R_h(0) = R_0 > 0$$

Where

$\frac{d\mathrm{PM}_{2.5}}{dt}$ represents the rate of change of particulate matter $\mathrm{PM}_{2.5}$.

$\mathrm{PM}_{2.5}$ represents the concentration of particulate matter $\mathrm{PM}_{2.5}$ over time.

α_1 represents the intrinsic growth rate parameter value for the particulate matter $\mathrm{PM}_{2.5}$.

β_1 represents the intra-competition coefficient of particulate matter $\mathrm{PM}_{2.5}$.

γ_1 represents the inter competition coefficient of the relative humidity amount to inhibits the growth of the relative humidity.

$\frac{dR_h(t)}{dt}$ represents the rate of change in relative humidity.

R_h represents the amount (%) of relative humidity over time.

α_2 represents the intrinsic growth rate parameter values for the relative humidity amount.

β_2 represents the intra-competition coefficient of relative humidity.

γ_2 represents the inter-competition coefficient of the concentration of the particulate matter $\mathrm{PM}_{2.5}$ to inhibits the growth of the relative humidity amount.

$\mathrm{PM}_{2.5}(0)$ represents the initial condition of the particulate matter $\mathrm{PM}_{2.5}$.

$R_h(0)$ represents the initial condition of the relative humidity amount.

Determination of Steady-State Solutions of the First-Order Model of the Interacting Environmental Variables

To study the qualitative behavior of the steady-state solution of the relative humidity and particulate pollutant level $PM_{2.5}$ interaction model, we had to derive from the dynamical system a unique positive co-existence steady-state solutions of the variables. From equation (1) we have:

$$\frac{dPM_{2.5}}{dt} = PM_{2.5}[\alpha_1 - \beta_1 PM_{2.5} - \gamma_1 R_h]\;\;,\; PM_{2.5}(0) = PM_{2.5_0} > 0$$
$$\frac{dR_h}{dt} = R_h[\alpha_2 - \beta_2 R_h - \gamma_2 PM_{2.5}]\;,\quad R_h(0) = R_0 > 0$$

To study the steady-state solutions of the system and its qualitative behavior.

At a steady-state solution,

$$\frac{dPM_{2.5}}{dt} = 0 \quad \text{and} \quad \frac{dR_h}{dt} = 0$$

Thus,

$$PM_{2.5}[\alpha_1 - \beta_1 PM_{2.5} - \gamma_1 R_h] = 0$$
$$R_h[\alpha_{21} - \beta_2 R_h - \gamma_2 PM_{2.5}] = 0$$

Suppose $(PM_{2.5_e}, R_e)$ is an arbitrary steady-state solution, then

$$PM_{2.5_e}[\alpha_1 - \beta_1 PM_{2.5_e} - \gamma_1 R_e] = 0$$
$$R_e[\alpha_{21} - \beta_2 R_e - \gamma_2 PM_{2.5_e}] = 0$$

Assuming that $PM_{2.5_e} \neq 0$ and $R_e \neq 0$, using Crammer's rule, we obtain

$$PM_{2.5_e}(\alpha_1 - \beta_1 PM_{2.5_e} - \gamma_1 R_e) = 0$$
$$R_e(\alpha_2 - \beta_2 R_e - \gamma_2 PM_{2.5_e}) = 0$$

$PM_{2.5_e} \neq 0$ and $R_e \neq 0$

$\alpha_1 - \beta_1 PM_{2.5_e} - \gamma_1 R_e = 0$

and

$\alpha_2 - \beta_2 R_e - \gamma_2 PM_{2.5_e} = 0$

In matrix form

$\beta_1 PM_{2.5_e} + \gamma_1 R_e = \alpha_1$

$\gamma_2 PM_{2.5_e} + \beta_2 R_e = \alpha_2$

$$\begin{pmatrix} \beta_1 & \gamma_1 \\ \gamma_2 & \beta_2 \end{pmatrix} \begin{pmatrix} PM_{2.5_e} \\ R_e \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

Where $Ax = b$

Let $H = \begin{pmatrix} \beta_1 & \gamma_1 \\ \gamma_2 & \beta_2 \end{pmatrix}$, $\det H = |H| = \begin{vmatrix} \beta_1 & \gamma_1 \\ \gamma_2 & \beta_2 \end{vmatrix}$

$|H| = \beta_1 \beta_2 - \gamma_1 \gamma_2$

Let $\Delta PM_{2.5_e} = \begin{pmatrix} \alpha_1 & \gamma_1 \\ \alpha_2 & \beta_2 \end{pmatrix}$, $\det \Delta PM_{2.5_e} = |\Delta PM_{2.5_e}| = \begin{vmatrix} \alpha_1 & \gamma_1 \\ \alpha_2 & \beta_2 \end{vmatrix}$

$|\Delta PM_{2.5_e}| = \alpha_1 \beta_2 - \gamma_1 \alpha_2$

Let $\Delta R_e = \begin{pmatrix} \beta_1 & \alpha_1 \\ \gamma_2 & \alpha_2 \end{pmatrix}$, $\det \Delta R_e = |\Delta R_e| = \begin{vmatrix} \beta_1 & \alpha_1 \\ \gamma_2 & \alpha_2 \end{vmatrix}$

$|\Delta R_e| = \beta_1 \alpha_2 - \gamma_2 \alpha_1$

$PM_{2.5_e} = \frac{|\Delta PM_{2.5_e}|}{|H|} = \frac{\alpha_1 \beta_2 - \gamma_1 \alpha_2}{\beta_1 \beta_2 - \gamma_1 \gamma_2}$

$$R_e = \frac{|\Delta A_e|}{|H|} = \frac{\beta_1\alpha_2 - \gamma_2\alpha_1}{\beta_1\beta_2 - \gamma_1\gamma_2}$$

Hence, the positive co-existence steady-state solution is provided.

$$\left[PM_{2.5_e}, R_e\right] = \left[\frac{\alpha_1\beta_2 - \gamma_1\alpha_2}{\beta_1\beta_2 - \gamma_1\gamma_2}, \frac{\beta_1\alpha_2 - \gamma_2\alpha_1}{\beta_1\beta_2 - \gamma_1\gamma_2}\right]$$

$$\alpha_1\beta_2 - \gamma_1\alpha_2 > 0, \quad \beta_1\alpha_2 - \gamma_2\alpha_1 > 0, \quad \beta_1\beta_2 - \gamma_1\gamma_2 > 0$$

This assumption is purely based on the fact that the amount of pollutant level $PM_{2.5}$ and meteorological variables in an ecosystem are purely non-negative dependent variables.

For the purpose of this study, we have used the method of p-vector norms with respect to the 1-norm, 2-norm and infinity-norm to obtain best fit model parameter values which has small error used for the prediction of the interaction between particulate matter $PM_{2.5}$ and relative humidity environmental variables.

Existence of solutions for initial value problems

This subsection deals with qualitative analysis related to the solution of the initial-value problems for ordinary differential equations. Referring to the above class of mathematical formulations, the following definitions can be given:

Well-Posedness:

A problem is well formulated if the evolution equation is associated with the correct number of initial (or boundary) conditions for its solution, while a problem is well posed if the solution exists, it is unique and depends continuously on the initial data. The main purpose of a model related to a certain physical system is to predict, for a certain time interval, the behavior of the system starting from the knowledge of the state at the initial time t_0. The predictions of the model are then obtained by solving the initial-value problem. To do that, there are some basic requirements that a problem should satisfy:

i) The solution should exist at least for the period of time desired.

ii) The solution should be unique.

iii) The solution should depend continuously on the initial data and on the parameters of the model so that if a small error is made in describing the present state, one should expect the effect of this error to be small in the future. As already stated, if these requirements are satisfied, then the initial-value problem is said to be well posed.

Numerical Iterations Mathematical Preliminaries

Following Akpodee (2019), when numerical solutions to initial value problems (IVPs) are required that cannot be obtained by analytical means, it is necessary to use numerical methods. From the numerical methods that exist in solving initial value problems, we have only considered the popular fourth-order Runge-Kutta method in this study as part of the mathematical preliminaries. The mathematical structure and the theoretical definitions of this method are presented as follows:

The fourth-order Runge-Kutta (R-K) method is an accurate and flexible method based on a Taylor series approximation to the function $f(x, y)$ in the initial value problem

$$\frac{dy}{dx} = f(x, y)$$

Subject to the initial condition $y\ (x_o) = y_o$

The increment *h* in *x* may be changed at each step, but is usually kept constant so that after *n* steps, we have

$$x_n = x_o + nh$$

The Runge-Kutta algorithm for the determination of the approximation y_{n+1} to $y(x_{n+1\,)})$ is

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where,

$$k_1 = hf(x_{n,}y_{n,})$$
$$k_2 = hf(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}k_1)$$
$$k_3 = hf(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}k_2)$$
$$k_4 = hf(x_{n+1,}y_n + k_3)$$

The local error involved in the determination of y_{n+1} from y_n is $0(h^5)$

The above method can be extended to find solution to a system of differential equations such as

$$\frac{dy}{dx} = f(x, y, z)$$

$$\frac{dz}{dx} = g(x, y, z)$$

Subject to the initial condition $y\ (x_o) = y_o$ and $z\ (x_o) = z_o$

These are the types of equations considered by this study which consists of a system of two first order nonlinear differential equations.

At the *nth* integration step, using a step of length *h*, the Runge-Kutta Algorithm for the system takes the form

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$z_{n+1} = z_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

Where,

$$k_1 = hf(x_{n,}y_{n,}z_n)$$
$$k_2 = hf(x_n + \frac{1}{2}h\,,\ y_n + \frac{1}{2}k_1,\ z_n + \frac{1}{2}K_1)$$
$$k_3 = hf(x_n + \frac{1}{2}h\,,\ y_n + \frac{1}{2}k_{2,}z_n + \frac{1}{2}K_2)$$
$$k_4 = hf(x_n + h\,,\ y_n + k_3,\ \ z_n + K_3)$$

and

$$K_1 = hg(x_{n,}y_{n,}z_n)$$
$$K_2 = hg(x_n + \frac{1}{2}h\,,\ y_n + \frac{1}{2}k_{1,}z_n + \frac{1}{2}K_1)$$
$$K_3 = hg(x_n + \frac{1}{2}h\ ,\ y_n + \frac{1}{2}k_2,\ z_n + \frac{1}{2}K_2)$$
$$K_4 = hg(x_n + h\,,\ y_n + k_3 z_n + K_3)$$

As with the Runge-Kutta method, the local error involved in the determination of y_{n+1} from y_n and z_{n+1} from z_n is $0(h^5)$

It is a good numerical analysis practice that in the event of a complex dynamical system that can not admit an analytic solution for impact and sensitivity analysis of pollutant level $PM_{2.5}$ interactions with meteorological variables, we have to adopt an alternative method to study the qualitative characteristics of such interactions. This is a challenging environmental problem that will be tackled computationally using MATLAB ODE45 numerical scheme.

In this study, the submission and rationale that underpin the choice of numerical simulation approach (Iterations) for the pollutant level $PM_{2.5}$ interactions with meteorological variables mathematical model formulation are stated as follows:

i. The closed-form solution (Exact solution by direct integration) maybe very complicated and difficult to evaluate analytically because of its complex environmental complication.

ii. There is no other choice which is to say that in such situation or scenario, no closed-form solution can be found.

iii. Computational approach required a large number of steps (Iterations) with chosen small step size so that a great deal of round off error can be accumulated.

iv. This approach will enhance multiple precision computations on our predictions and projections.

v. The meteorological variables relative abundance/ pollutant level $PM_{2.5}$ stock can be estimated accurately because of its role in climate change and also its role in environmental decision and policy making by environmental protection agencies.

The parameters are stated as follows for $PM_{2.5}$-RH Interaction:

alpha1=0.6044190656; beta1=0.0266852097; r1=0.0001342; alpha2=0.04725; beta2=0.00063084; r2=0.000111;

Results and Discussions.

Table 1: Impact of 50% variation of the intrinsic growth rate of $PM_{2.5}$ in the interaction between $PM_{2.5}$ and Relative Humidity RH.

Here, quantification of the impact of a 50% variation of the intrinsic growth rate of $PM_{2.5}$ interacting with relative humidity for a time interval 0(30)360 in months was studied. The decreased in the intrinsic growth rate parameter of $PM_{2.5}$ captured grey areas where there are depletions and gains in the coordinates of the interacting variables and its impact on the environment in terms of percentage quantification.

Figure 1: Solution trajectory of the impact of 50% variation of the intrinsic growth rate of $PM_{2.5}$ and RH interaction against time for an interval of 0(30)360 days.

Table 2: **Impact of 150% variation of the intrinsic growth rate of $PM_{2.5}$ in the interaction between $PM_{2.5}$ and Relative Humidity RH.**

Here, quantification of the impact of a 150% variation of the intrinsic growth rate of $PM_{2.5}$ interacting with relative humidity for a time interval 0(30)360 in months was studied. The increased in the intrinsic growth rate parameter of $PM_{2.5}$ captured grey areas where there are depletions and gains in the coordinates of the interacting variables and its impact on the environment in terms of percentage quantification.

Figure 2 Solution trajectory of the impact of 150% variation of the intrinsic growth rate of $PM_{2.5}$ and RH interaction against time for an interval of 0(30)360 days

Table 1: Impact of 50% variation of the intrinsic growth rate of $PM_{2.5}$ in the interaction between $PM_{2.5}$ and Relative Humidity RH

Time	(Months) N1($PM_{2.5}$)	N11(PM2.5)m	EPD	N2(Rh1)	N21(Rh2)m	EPG
0	19.4000	19.4000	0	62.0000	62.0000	0
29.0000	22.3044	10.9791	50.7760	68.2886	69.6653	2.0160
58.0000	22.2969	10.9604	50.8434	70.2219	72.0704	2.6324
87.0000	22.2912	10.9648	50.8112	70.7694	72.7324	2.7738
116.0000	22.2819	10.9568	50.8262	70.9204	72.9086	2.8035
145.0000	22.2899	10.9608	50.8263	70.9619	72.9554	2.8092
174.0000	22.2966	10.9583	50.8522	70.9733	72.9676	2.8099
203.0000	22.3111	10.9565	50.8919	70.9766	72.9708	2.8096
232.0000	22.2932	10.9602	50.8362	70.9772	72.9718	2.8101
261.0000	22.2917	10.9537	50.8619	70.9774	72.9718	2.8098
290.0000	22.2973	10.9619	50.8377	70.9776	72.9721	2.8101
319.0000	22.2814	10.9537	50.8391	70.9774	72.9719	2.8100
348.0000	22.2917	10.9626	50.8221	70.9775	72.9722	2.8102

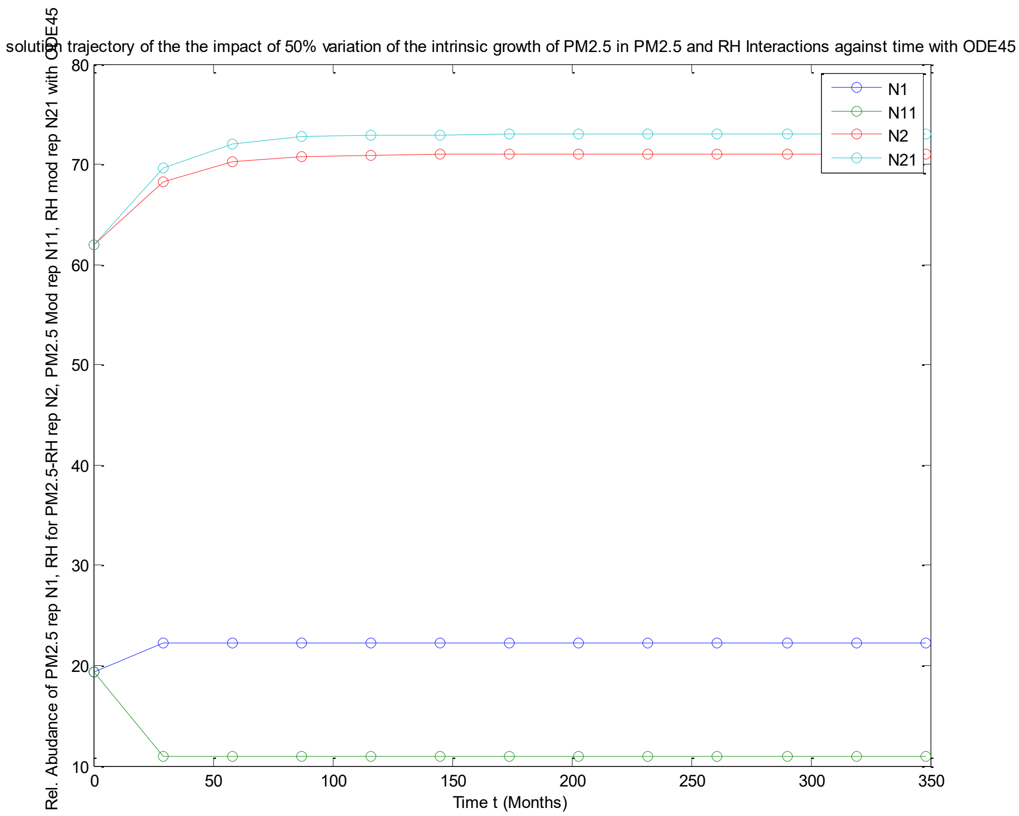

Figure 4.2: Solution trajectory of the impact of 50% variation of the intrinsic growth rate of $PM_{2.5}$ and RH interaction against time for an interval of 0(30)360 days

Table 4.4: Impact of 150% variation of the intrinsic growth rate of $PM_{2.5}$ in the interaction between $PM_{2.5}$ and Relative Humidity RH

Time(Months)	N1($PM_{2.5}$)	N11($PM_{2.5}$)m	EPD	N2(Rh1)	N21(Rh2)m	EPG
0	19.4000	19.4000	0	62.0000	62.0000	0
29.0000	22.3044	33.6432	50.8361	68.2886	66.8984	2.0358
58.0000	22.2969	33.6365	50.8568	70.2219	68.3792	2.6241
87.0000	22.2912	33.6317	50.8742	70.7694	68.8109	2.7675
116.0000	22.2819	33.6268	50.9155	70.9204	68.9342	2.8006
145.0000	22.2899	33.6250	50.8532	70.9619	68.9692	2.8082
174.0000	22.2966	33.6275	50.8188	70.9733	68.9791	2.8098
203.0000	22.3111	33.6287	50.7264	70.9766	68.9820	2.8103
232.0000	22.2932	33.6261	50.8355	70.9772	68.9827	2.8100
261.0000	22.2917	33.6206	50.8211	70.9774	68.9829	2.8101
290.0000	22.2973	33.6201	50.7811	70.9776	68.9830	2.8102
319.0000	22.2814	33.6279	50.9239	70.9774	68.9831	2.8098
348.0000	22.2917	33.6199	50.8183	70.9775	68.9830	2.8101

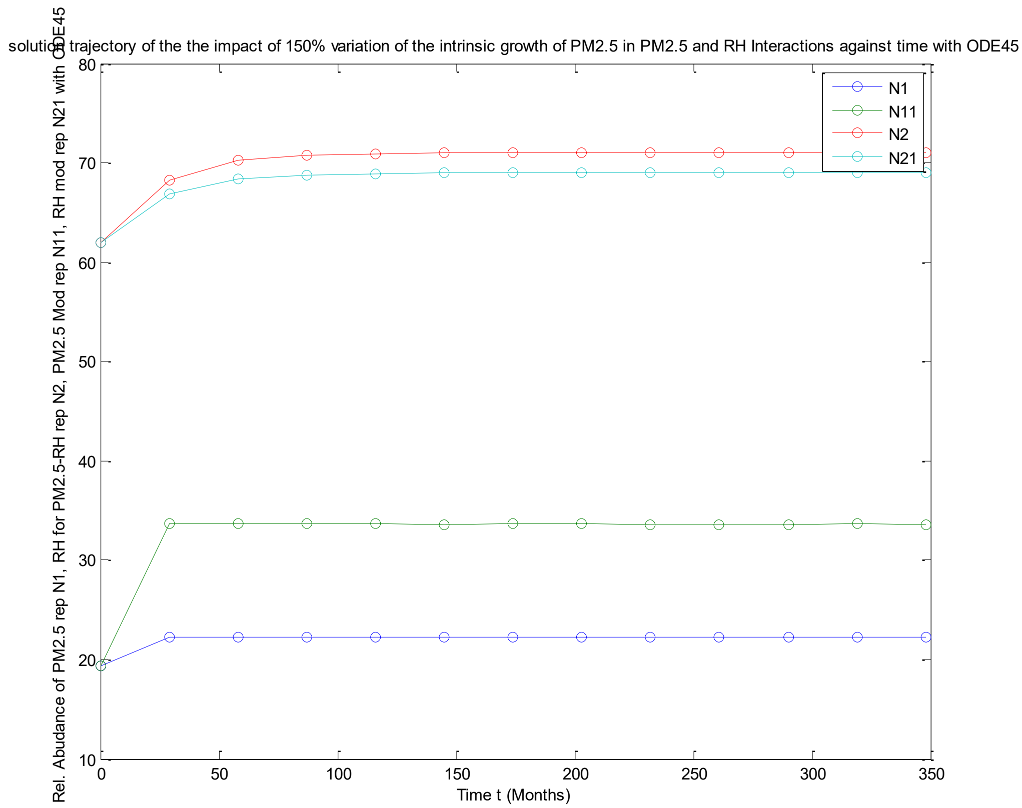

Figure 4.3: Solution trajectory of the impact of 150% variation of the intrinsic growth rate of $PM_{2.5}$ and RH interaction against time for an interval of 0(30)360 days

Discussion on the Impact of 50% variation of the intrinsic growth rate of $PM_{2.5}$ in the interaction between $PM_{2.5}$ and Relative Humidity RH

In quantifying the impact of a 50% variation of the intrinsic growth rate of $PM_{2.5}$ interacting with relative humidity, we observed that on the base day of our experimental time, the initial value here called initial condition (IC) when all model parameter values are fixed for both the pollutant level 2.5 and the relative humidity are recorded as 19.4 and 62.0 respectively in amount and further fluctuates to a converging saturated value of 22.2917 and 70.9775 respectively in amount at an optimal time of over 360 days.

Due to a 50% variation of the growth rate of the pollutant level 2.5 at an interval of 30days for a total time period of 366 days, we observed that the modified value of the $PM_{2.5}$ experienced a drastic depletion down the trend from an initial value of 19.40 and decreased to a converging saturated value of 10.9626 in amount over an optimal time of 360days with the relative humidity having a slight gain from an initial value of 62.0 which increases to a converging saturated value of 72.9722 in amount over an optimal time of 360days when compared to the fixed coordinates and the quantification which measures this severe depletion and slight gain were also computed as well.

In this scenario, we observed that on the base day of the experimental time, there was no impact as the quantification measured were recorded as zero (0) for both coordinates respectively. However, from the 1st 30days up to 366 days, the quantification for the $PM_{2.5}$ ranges from 50.7760 percent to a converging value of 50.8221 percent, and that of relative humidity ranges from 2.0160 percent to a converging value of 2.8102 percent.

In terms of environmental studies and modeling, these quantification shows that a severe depletion in the pollutant level 2.5 will serve as a good information since the environment will be relieved of the pollutant and that of the slight gained in the relative humidity indicates a hospitable environment and conducive for ecosystem functioning. The trend of the solution trajectory of this qualitative behaviour showed clearly their various optimal characteristics due to 50% percentage decreased of the $PM_{2.5}$ as well as their converging saturated values of the modified coordinates with expected percentage effects in quantifications. This information is vital for various pollution and meteorological agencies and will serves as a guide for environmental and ecosystem functioning.

Discussion on the Impact of 150% variation of the intrinsic growth rate of $PM_{2.5}$ in the interaction between $PM_{2.5}$ and Relative Humidity RH

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Due to a 150% variation of the growth rate of the pollutant level 2.5 at an interval of 30days for a total time period of 366 days, we observed that the modified value of the $PM_{2.5}$ experienced a strong gained down the trend from an initial value of 19.40 and increases to a converging saturated value of 33.6199 in amount over an optimal time of 360days with the relative humidity having a slight depletion from an initial value of 62.0 which decreases to a converging saturated value of 68.9830 in amount over an optimal time of 360days when compared to the fixed coordinates and the quantification which measures this severe depletion and slight gain were also computed as well.

In this scenario, we observed that on the base day of the experimental time, there was no impact as the quantification measured were recorded as zero (0) for both coordinates respectively. However, from the 1st 30days up to 366 days, the quantification for the $PM_{2.5}$ ranges from 50.8361 percent to a converging value of 50.8183 percent, and that of relative humidity ranges from 2.0358 percent to a converging value of 2.8101 percent.

In terms of environmental studies and modeling, these quantification shows that a severe gained in the pollutant level 2.5 will serve as a good information since the environment will be vulnerable of the pollutant concentration as well as high risk of health exposure and hostility with the slight depletion in the relative humidity indicates an inhospitable environment and unconducive for ecosystem functioning. The trend of the solution trajectory of this qualitative behaviour showed clearly their various optimal characteristics due to 50% percentage increase of the $PM_{2.5}$ as well as their converging saturated values of the modified coordinates with expected percentage effects in quantifications. This information is vital for various pollution and meteorological agencies and will serves as a guide for environmental and ecosystem functioning

Summary of Environmental Implications of Results Discussions

Low relative humidity can be influenced by several factors related to suspended particulate matter, particularly PM_1 (particulate matter with a diameter of 1 micrometer or less) and $PM_{2.5}$ (particulate matter with a diameter of 2.5 micrometers or less). Here are some key factors:

1. **Aerosol Properties**: PM_1 and $PM_{2.5}$ can affect humidity levels due to their ability to absorb and scatter water vapor. Smaller particles can have a higher surface area-to-volume ratio, which can influence their hygroscopic properties, potentially leading to lower humidity levels in the surrounding air.
2. **Source of Particulates**: The origin of PM1 and PM2.5, such as industrial emissions, vehicle exhaust, or natural sources like wildfires, can impact their chemical composition. Some particles may be less hygroscopic, meaning they do not attract water vapor effectively, contributing to lower relative humidity.
3. **Atmospheric Conditions**: Weather conditions, including temperature and pressure, play a significant role in relative humidity. High temperatures can increase the capacity of air to hold moisture, while the presence of particulate matter can alter local microclimates, potentially leading to lower humidity levels.
4. **Urbanization and Land Use**: Urban areas often have higher concentrations of PM1 and PM2.5 due to human activities. The heat island effect in cities can lead to increased temperatures, which may lower relative humidity levels.

5. **Chemical Reactions**: Particulate matter can participate in chemical reactions in the atmosphere that may consume water vapor or produce compounds that do not retain moisture, further contributing to lower humidity.
6. **Transport and Dispersion**: The movement of air masses can influence humidity levels. If PM_1 and $PM_{2.5}$ are transported from areas with low humidity, they can bring those conditions to new locations, affecting local humidity levels.

Understanding these factors is crucial for addressing air quality and its impacts on health and the environment.

High relative humidity can be influenced by several factors related to suspended particulate matter, particularly PM1 (particulate matter with a diameter of 1 micrometer or less) and $PM_{2.5}$ (particulate matter with a diameter of 2.5 micrometers or less). Here are some key factors:

1. **Aerosol Properties**: PM1 and $PM_{2.5}$ can act as cloud condensation nuclei (CCN), which are essential for the formation of clouds and precipitation. The presence of these particles can enhance the ability of water vapor to condense, leading to increased humidity levels.
2. **Chemical Composition**: The chemical makeup of particulate matter can affect its hygroscopicity, or its ability to attract and hold water. Particles that contain salts, sulfates, or organic compounds can absorb moisture from the air, contributing to higher relative humidity.
3. **Temperature and Pressure**: Higher temperatures can increase the capacity of air to hold moisture. When PM1 and PM2.5 are present in warmer conditions, they can facilitate the retention of moisture, leading to elevated humidity levels.
4. **Urbanization and Pollution**: Areas with high levels of PM_1 and $PM_{2.5}$, often due to urbanization and industrial activities, can experience localized increases in humidity. The combination of heat from urban surfaces and the presence of particulate matter can create microclimates with higher humidity.
5. **Meteorological Conditions**: Weather patterns, such as low-pressure systems and stagnant air, can lead to the accumulation of particulate matter and moisture in the atmosphere, resulting in higher relative humidity.
6. **Vegetation and Land Use**: Areas with significant vegetation can influence humidity levels through transpiration, which can interact with particulate matter in the air, potentially increasing humidity.

Understanding these factors is crucial for addressing air quality and its impacts on health and the environment.

Impact of relative humidity on suspended particulate matter PM_1 and $PM_{2.5}$

Relative humidity (RH) has a significant impact on suspended particulate matter, particularly PM1 (particulate matter with a diameter of 1 micrometer or less) and $PM_{2.5}$ (particulate matter with a diameter of 2.5 micrometers or less). Here are some key effects:

1. **Hygroscopic Growth**: Many particulate matter components, especially those containing salts and other hygroscopic materials, can absorb moisture from the air when relative humidity is high. This leads to an increase in the size of the particles, which can affect their behavior in the atmosphere, including their ability to scatter light and their deposition rates.
2. **Cloud Condensation Nuclei (CCN)**: At higher relative humidity levels, PM_1 and $PM_{2.5}$ can act as cloud condensation nuclei, facilitating cloud formation and influencing precipitation processes. This can lead to changes in local weather patterns and affect air quality.
3. **Chemical Reactions**: High humidity can enhance chemical reactions involving particulate matter. For example, moisture can facilitate the formation of secondary organic aerosols (SOAs) and other compounds, which can alter the composition and toxicity of PM_1 and $PM_{2.5}$.
4. **Transport and Dispersion**: Relative humidity can influence the transport and dispersion of particulate matter. High humidity can lead to the aggregation of particles, making them heavier and potentially causing them to settle more quickly. Conversely, low humidity can keep particles suspended in the air for longer periods.
5. **Health Implications**: The size and composition of PM_1 and $PM_{2.5}$ can change with varying humidity levels, which may affect their inhalation and deposition in the respiratory system. Changes in particle size and composition can influence their toxicity and health impacts.
6. **Visibility and Aesthetics**: High humidity can lead to increased visibility issues due to the scattering of light by larger, hygroscopic particles. This can affect air quality perceptions and have implications for tourism and outdoor activities.
7. **Seasonal Variations**: Relative humidity often varies seasonally, which can lead to fluctuations in PM1 and $PM_{2.5}$ concentrations. For instance, during the summer months, higher humidity levels may lead to increased aerosol formation, while winter months may see lower humidity and different particulate matter dynamics.

Understanding the relationship between relative humidity and suspended particulate matter is crucial for air quality management, climate modeling, and public health assessments.

Conclusion

The key results that this work have achieved are stated as follows:

1) The Impact of 50% variation of the intrinsic growth rate of $PM_{2.5}$ in the interaction between $PM_{2.5}$ and Relative Humidity RH.
2) The Impact of 150% variation of the intrinsic growth rate of $PM_{2.5}$ in the interaction between $PM_{2.5}$ and Relative Humidity RH.
3) Solution trajectory of the Impact of 150% variation of the intrinsic growth rate of $PM_{2.5}$ in the interaction between $PM_{2.5}$ and Relative Humidity RH against time for an interval of 0(30)360 days.
4) Solution trajectory of the Impact of 150% variation of the intrinsic growth rate of $PM_{2.5}$ in the interaction between $PM_{2.5}$ and Relative Humidity RH against time for an interval of 0(30)360 days.

Recommendations

This study work will recommend the following open research problem namely:

1.) The impact of the variation of the intra-competition coefficient of the pollutant level 2.5 and relative humidity together.
2.) The impact of the variation of the inter-competition coefficient of the pollutant level 2.5 and relative humidity together.
3.) The impact of changing initial condition of the pollutant level 2.5 and relative humidity together.
4.) The parameter estimation of other pollutants and meteorological variables over time.
5.) The impact of a mild and severe random perturbation on the pollutant level 2.5 and relative humidity together for two scenarios.

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