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Polynomial Model for Determination of Local Geoid: A Case Study of University of Maiduguri, Borno State, Nigeria

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Abstract

Conversion of ellipsoidal heights of Global Navigation Satellite System (GNSS) observations to orthometric heights for efficient use requires an accurate determination of the local geoid which is essential for myriad of applications, ranging from precise positioning to other geodetic studies. However, local geoid has not been adequately determined in several locations globally. This thesis presents a comprehensive study on the development and application of a polynomial model for the determination of the local geoid, with a specific focus on the University of Maiduguri, located in Borno State, Nigeria. The research reviewed of existing geodetic methods and models, highlighting the limitations and challenges faced in the local context. Then, an innovative polynomial model is proposed and developed to address the specific geoid determination requirements of the study area. The model integrates ground-based measurements, satellite data, and advanced computational techniques to enhance accuracy and reliability. The methodology employed involves an extensive data collection campaign, incorporating precise leveling data, and GNSS observations. The collected data are processed and analyzed to derive the coefficients of the polynomial model, tailored to the local characteristics of the University of Maiduguri region. The research used rigorous least square technique, to compute eleven coefficients for the model, by Least squares adjustment procedure in Matlab environment. The result from the adjusted geoid models to the survey data at 31 points revealed that the mean accuracy of local geoidal heights at 0.026, and EGM20 - 5.624m level was attainable The study revealed the effectiveness of the proposed polynomial model in accurately determining the local geoid, providing valuable insights into the geodetic landscape of Borno State. The results were validated through comparisons with existing geoid models and ground truth measurements, demonstrating the model's robustness and applicability. This research contributes to the advancement of geodetic techniques in the specific context of Borno State, Nigeria, and lays the groundwork for further studies in geoid modeling and its applications in regional geosciences is hereby recommended.

Key words: polynomial model, local geoid, ellipsoidal heights, orthometric heights, GPS/Levelling.

Introduction

The accurate determination of reference surface (datum) such as height is an important component for determining the position on or below the earth surface (Kallio, Klügel, Marila, Mähler, Poutanen, & Saari, 2022). Determination of this surface has not easy; it has been the problems of geodesists in Africa as well as the developed world. Levelling is a process through which height of points or differences in elevation are determined reference to a surface (Aleem, Adesoye, & Bankole, 2016). Height is one-dimensional coordinate system used to define the metric elevated distance of a point from a reference surface to the point of interest (Odumosu, Ajayi, Idowu, & Adesina, 2018).Therefore, the knowledge of heights in real-life situation cannot be overemphasized; especially the relative positions of neighbouring points in a common reference surface. Hence, this shows that, any project involves infrastructural development such as: building constructions, skyscrapers, roads, bridges, dams, drainages, airports, tunnels, pipelines, are all dependents on height with accurate reference surface use. Therefore, the need for height in all aspect of human activities is essential (Kallio, *et al.,* 2022).

Geoid is the equipotential surface of the Earth's gravity field, which coincides with the average surface of the ocean and referred to as the Mean Sea Level (MSL), which is achievable in 18.6 years (Aleem, *et al.,* 2016). Orthometric height can be determined in two ways; direct and indirect methods. Direct method involves determination of height using spirit levelling (Aleem, 2013). The indirect method involves the use of Global Navigation Satellite System (GNSS) to determine ellipsoidal height for model computation of geoidal undulation. The indirect method is of two approaches, namely: absolute geoid model for computing absolute values of the geoidal undulation; and relative geoid model for computing geoidal undulation relative to the existing geoid model.

Hence, the problem of geodesy today is the precise determination of geoid. This issue has attracted the attentions of geodesists more than 200 years ago. When Gabriel Stroke started the solution in 1849 and Helmet in 1880, coupled with advent of satellite-based positioning techniques (GNSS) especially Global Positioning System (GPS). However, GNSS determined positions with reference to geocentric ellipsoid called World Geodetic System 1984 (WGS'84). However, GPS ellipsoidal heights are generally accepted and mostly used in engineering works because of its effortlessness of obtaining heights. While orthometric heights are reference to geoid, and determined with spirit leveling, these heights are practical and accurate than the theoretical ellipsoid heights. But difficult to obtained because it is tedious, laborious, and time-consuming over long distances. In order to utilize the opportunities of GPS techniques, the need to convert ellipsoidal heights into orthometric heights is necessary. Unfortunately, the geoid height for Nigeria has not been accurately determined because of lack of geoid model and adjustment of the network was by the development method rather than the correct projection method. Hence, observations are reduced to the geoid rather than the ellipsoid. Consequently, these geodetic reductions introduce distortions in the network (Uzodinma, 2005). Thus, it is essential to determine the local geoid in Nigeria for geodetic and engineering use.

All surveying activities are carried out on three basic surfaces referred as "geodetic surfaces", namely; the topographic (earth) surface, the geoid and the ellipsoid as presents in Figure 1

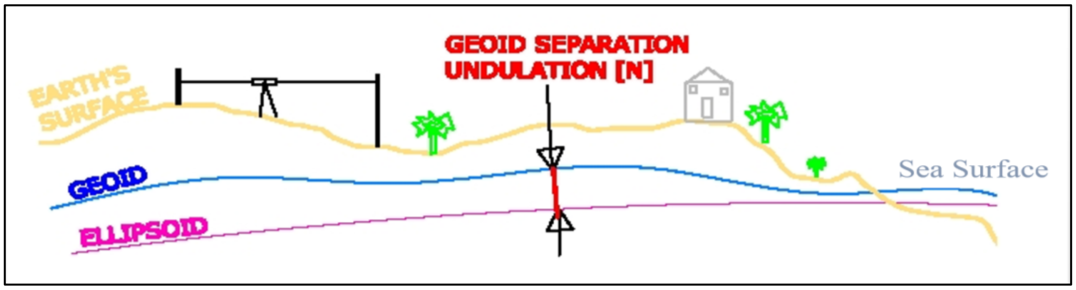


Figure 1: The three geodetic surfaces

$$N = h - H \quad (1)$$

Where;

N = Geoidal Undulation, **h** = Ellipsoidal height and **H** = Orthometric Height.

The vertical separation between the ellipsoid and the geoid is known as geoidal separation **(N)**. The geoid modelling use in this research is based on the three geodetic surfaces as;

The topographical surface, also known as the Earth's surface, is the tangible surface of the land and sea. This is the physical surface, where all activities; measurements and observations are made. But, this surface is irregular in nature, characterised by mountains, valleys, spurs, dunes, rivers, sea, and other features. Hence, geodetic computation cannot be done on this surface. The surface close to it is the geoid, which is smoother than the physical Earth surface despite its global undulations (changes). It serves as reference surface for orthometric height and is very close to the shape of an ellipsoid of revolution, but irregular because of the bulge. The ellipsoid is a mathematical surface approximating the physical reality of the earth while simplifying the geometry. It is a surface whose plane sections are all ellipses. This is the regular surface among the three geodetic surfaces; it has a regular shape which makes it possible for mathematical computations to be done on it. The ellipsoid serves as a basis for the 3D coordinates of satellite systems such as the GNSS, which is with reference to World Geodetic System 1984 (WGS 84) ellipsoid.

Determination of local geoid has attracted the attention of researchers in areas of Science and Geodesy for centuries. Hence, many researchers have used different approaches for geoid modelling, which include; gravimetric, astro-gravimetric, astrogeodetic geometric, and earth geopotential model. These are absolute geoid models which determine the magnitude of geoidal undulation as a function of positions. Also, there are empirical methods, which processed using numerical modelling of the geoid, such as, geometric methods and EGM.

Geometric (GPS/levelling) method was employed in this study to determine local geoid model for University of Maiduguri, using third order polynomial model because, is flexible and has

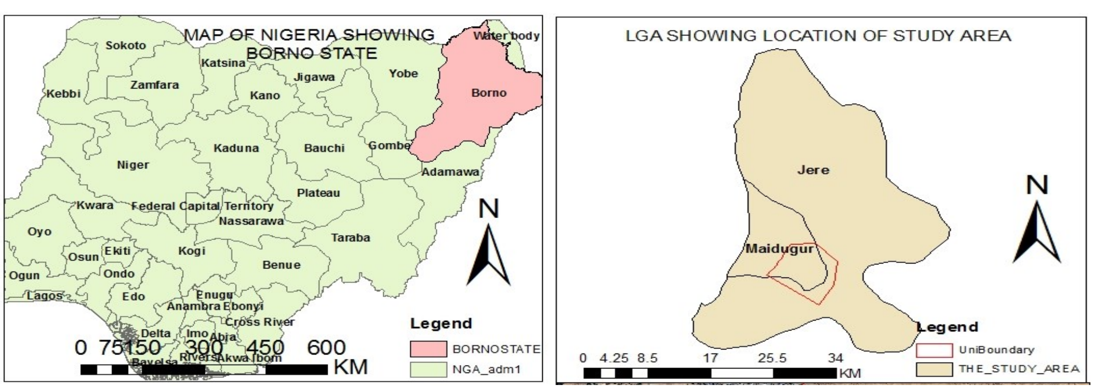
more coefficients than linear (two-coefficients) or quadratic (three-coefficients) polynomials, it allow more coefficients for precise fit to data model. It is efficiency in computation and evaluation, making it a good choice for large datasets, it model curvature more effectively which make it essential for capturing the complex shape of the geoid, to decide the extent of replacing leveling data with GPS observations.
University of Maiduguri is located in Maiduguri the capital of Borno State, its developing rapidly and massive constructions are currently ongoing such as buildings, roads, bridges, flyovers and power gas plants. Most of these observations are carried out using GNSS/GPS observation. Some of the preliminary surveys used for the design of these constructions were obtained using GNSS receiver in Real-Time Kinematic (RTK) mode. Ellipsoidal heights are used instead of orthometric heights. orthometric heights are essential for engineering as they are suitable for determining gradients to decide the direction of flow of water at different levels of the proposed constructions. This study tends to determine the best geoid model for study area by determining local geoid using polynomial interpolation method for University of Maiduguri.

Methodology

This section provides detailed description of the study area and the general overview of the methods and materials, the types of data and software used.

Study Area

University of Maiduguri is situated along Bama road in Maiduguri, Borno State Nigeria. The University lies between two local government areas of Jere and Maiduguri Metropolitan Council (MMC). It is geographically located between latitudes 11° 48' 7.57" and 11° 47' 33.10" north of the Equator and longitudes 13° 11' 49.49" and 13° 12' 11.00" east of the Greenwich meridian, and covers a total land area of about 1,804 km² (Figure 2). The weather in Maiduguri is typically hot and dry, with temperatures often reaching over 40°C (104°F) during the hottest months (April and May). The relief rainfall in the area is generally low, as Maiduguri is part of the semi-arid Sahel region. However, when rainfall does occur, it is often in the form of convective or relief rainfall, caused by the uplift of air over elevated terrain. The study area has an average elevation of approximately 300 meters (984 feet) above sea level, with fairly stable topography that ranges from 0.1 to 5 meters.



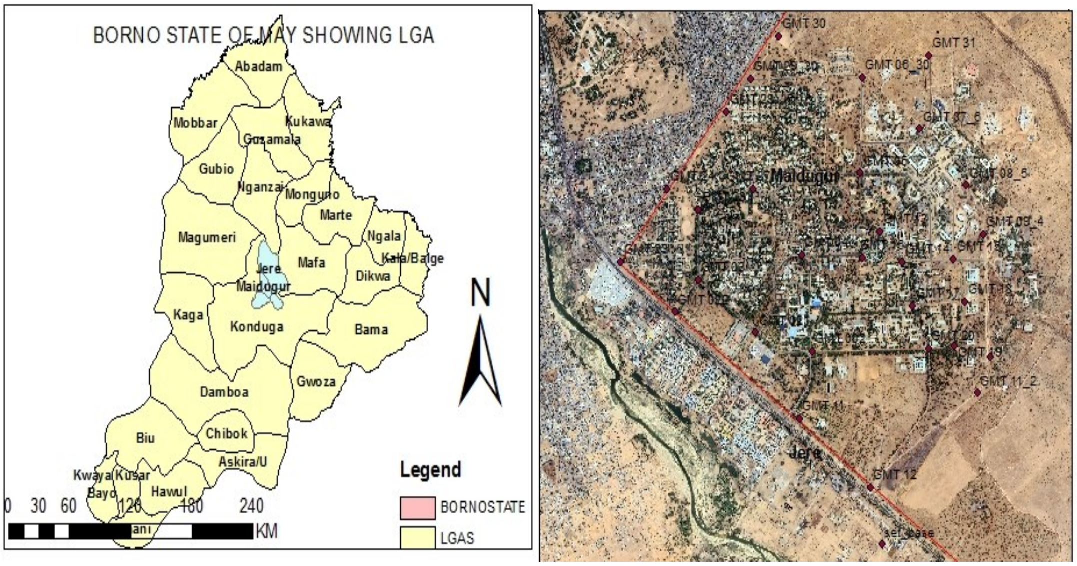


Figure 2: Map of the Study Area. Source: author

Materials and Methods

The materials and methods used in the study are as follows.

Materials Used

The materials used in this study include the instruments, hardware and software.

Instruments Used

The Instruments consists of the following:

i. Hi Target (DGPS V90plus) receiver with accessories
ii. Digital /Electronic Levelling instrument with accessories
iii. 100m Tape
iv. Hammer

Hardware and Software

The hardware and software consist of the following:

i. HP EliteBook8440 laptop computer
ii. Printer (Black and white/color printer)
iii. ArcGIS 10.8
iv. QGIS 3.481
v. Matlab R2023b
vi. ICGEM Online Global Geoid
vii. Google Earth Pro
viii. Microsoft word

Methods Used

The methods employed in the study are described in Fugure 3 below.

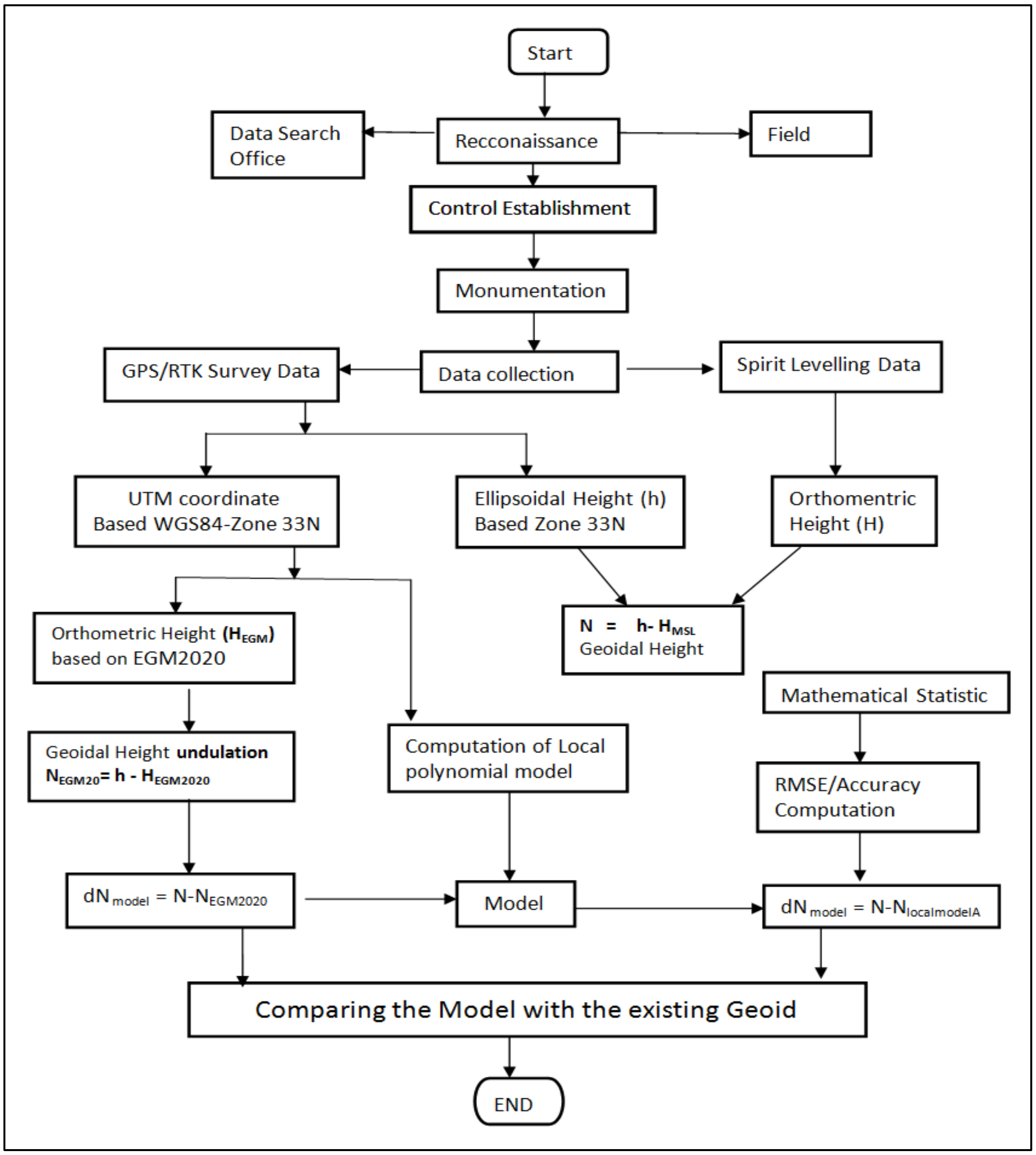


Figure 3: Flow chart of the methodology

Observation of ellipsoidal (h) and Orthometric Heights (H)

The field operation was set for observed coordinates and ellipsoidal heights using Hi-Target V90+ (DGPS) and electronic levelling instrument using Orthometric Height, distributed points in the study area. The total number of 31 points data acquired in the study. The points include one primary control station (BOS010) located along Bama Road, in front of industrial court.

GNSS/GPS Observation

GNSS observations were carried out using Hi-Target V 90plus dual-frequency receivers was used to obtained the coordinates (Easting's, Nothings and ellipsoidal heights) of 30 points, the observations were carried out relative to the control station BOS010 using the Real time kinematics (RTK) mode method of GNSS observation. The information of BOS010 was extracted from Borno Geographic Information Service BOGIS, the Office of the Surveyor General, Born State.

Levelling Observations

The levelling observation was carried out in accordance with Guidelines and specifications for control of Geodetic Surveys in Nigeria was follow to ensure that the levelling operation is consistent with geodetic standards (SURCON, 2003). A close leveling network were performed to check the quality of orthometric heights of existing reference points and also to calculate the heights of the newly established points. Therefore, the levelling Observations was carried out in three different loops, the first loop start from BOS010 which located in industrial court premises along Bama road; through GMT0012 to GMT001 and closed on BOS010. The second loop started from GMT010 through GMT012 to GMT014 and closed on GMT015. The third loop started from GMT015 GMT017 to GMT019 and closed on GMT024 (Figure 4).

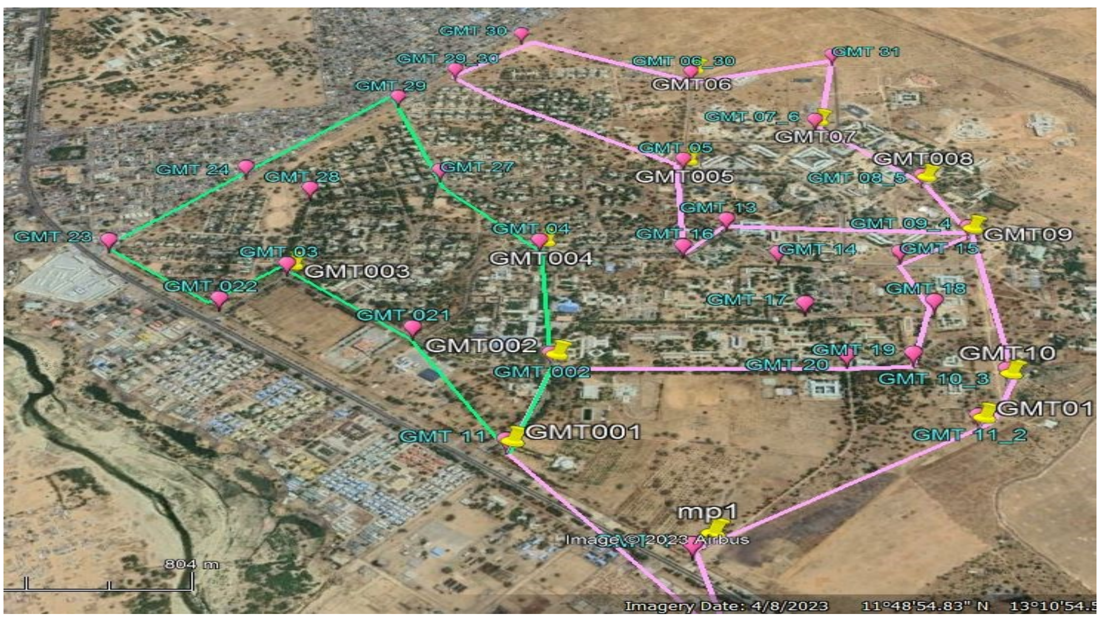


Figure 4: Google Earth imagery showing loops levelingof the

Computation of Geoidal Undulations (N) from ellipsoidal and Orthometric Heights

The ellipsoidal heights from GPS-RTK and orthometric heights of spirit leveling are substitute into above Equation 1 to calculate the values of the geoidal undulation.

Observation Equation Method of Least Squares Adjustment

Equations 2, 3, 4, 5, and 6 which are called observation equations, relate observed quantities to both observational residuals and independent unknown parameters. The functional relationship between adjusted observations and the adjusted parameters as given by Eteje and Oduyebo (2018) as:

$L_a = F(X_a)$ (2)

Where L_a = adjusted observations and X_a = adjusted parameters. Equation (2) is a linear function and the general observation equation models are obtained. The system of observation equations is presented by matrix notation (Eteje, *et al.,* 2018) as:

$V = AXL$ (3)

Where,

A = Design Matrix,

X = Vector of Unknowns

V = Residual Matrix

The residual, V is the difference between the estimates and the observed is usually useful when applying least squares adjustment technique for the determination of geoid heights interpolation model parameters, since it is equal to the difference between the known geoid heights and the computed geoid heights from GNSS observations and spirit levelling data.

L = AX (4)

Where, A = Design Matrix, X = Vector of Unknowns, L = Observation Matrix. That is,

(5)

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ m_n \end{pmatrix} \quad \text{and} \quad L = \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ m_n \end{pmatrix}$$

The determination of the unknown parameters, X, requires the normal matrix, N and the matrix of numeric terms, t to be deduced. It is to be noted here that the observations are not weighted. According to Eteje and Oduyebo (2018), a system of unweighted linear observation equations can be expressed in matrix notation as:

$A^T AX = A^T L$ (6)

To make X the subject of the formula, both sides of equation (6) will be divided by $A^T A$. Thus,

$X = (A^T A)^{-1} A^T L$ (7)

Equation 6 was used to compute the unknown/model parameter as give by (Oluyori, *et al.,*2018)

Where, $(A^T A)^{-1}$= Inverse of the normal matrix.

Polynomial Surface

Generally polynomial surface can be represents in different degree order therefore, this study has **special** third degree polynomial as a trend function to model a local geoid. The study, model the function of 2-D positions that is, easting and northing (x,y) of the control points, using polynomial model based on the chosen degree as shown in equation 2 below.

$$\boldsymbol{Model}, \boldsymbol{N = a_0 + a_1X + a_2Y + a_3X^2 + a_4Y^2 + a_5XY + a_6X^2Y + a_7XY^2 + a_8X^3 + a_9Y^3 + a_{10}Y^3X^3} \quad \textbf{(8)}$$

Where,

$y = ABS\ (Y - Y_o)$ (9)

$x = ABS\ (x - x_o)$

y = Northing coordinates of the Observed Station

x = Easting coordinates of the Observed Station

Y_o= Northing coordinates of the Origin (average of the northing coordinates)

x_o= Easting coordinates of the Origin (average of the easting coordinates)

Accuracy/Reliability of Geometric Geoid Model

The accuracy of the geometric geoid models is obtained using Root Mean Square Error (RMSE) index. The RMSE is applied to compute the accuracy of the model, the geoid heights of the selected points from the geoid model are compared with their corresponding known geoid heights to obtain the geoid height residuals. The geoid height residuals and the total number of the selected points are used to compute the RMSE as well as the accuracy of the model. The RMSE index for the computation of the geometric geoid model accuracy as given by Farsat, Heeto Abdulrahman (2021) as:

$$RMSE = \pm \sqrt{\frac{\sum_{i=0}^{n} \cdot (\Delta \mathbf{NV}_{LOCAL})^2}{n}} \quad (10)$$

Where

$\Delta NV_{LOCAL} = N_{KNOWN} - N_{LOCAL}$(Residual) (11)

N_{KNOWN} = Known geoid height of point

N_{LOCAL} = Computed model geoid height of point

n = number of points

Results and Discussion

The results of the study are presented and discussed below.

Observed ellipsoidal and orthometric heights

Therefore, the results of (31) points whose UTM coordinates, ellipsoidal heights and orthometric heights were obtained are given below (table 1).The table presents the lists of coordinates Eastings, Northings, and Ellipsoidal heights (h) of 31 points obtained by GPS-RTK mode and the reduced leveling orthometric heights (H) from Digital leveling field book.

Table 1: Coordinates obtained from GPS Observation and Geodetic Leveling

S/N	STATIONS	EASTINGS	NORTHINGS	ELLIPSOIDAL HEIGHT (h)m	ORTHOMETRIC HEIGHT (H)m
1	BSC_10	1304016.161	304094.0624	335.1683	319.27
2	GMT 001	1304361.762	304012.2762	331.9368	316.023
3	GMT 002	1304923.998	304767.4767	322.2108	306.305
4	GMT 003	1305140.850	304863.0222	321.4848	305.61
5	GMT 004	1305879.820	304815.4248	319.3148	303.45
6	GMT 005	1306177.738	304695.4996	318.6078	302.77

7	GMT 006	1306518.011	304374.0706	319.0708	302.248
8	GMT 007	1306963.373	304443.493	318.0498	302.235
9	GMT 008	1306831.569	303971.2997	318.6718	302.765
10	GMT 009	1306256.765	303956.1915	320.8228	304.416
11	GMT 010	1305902.069	304091.7288	320.0298	304.29
12	GMT 011	1305749.021	303963.8282	321.6768	305.297
13	GMT 012	1305766.624	303538.7942	323.3268	307.393
14	GMT 013	1305719.455	304244.4862	320.5218	304.834
15	GMT 014	1305734.865	304603.3232	320.0548	304.19
16	GMT 015	1305474.904	304686.8112	320.3478	304.452
17	GMT 016	1305453.189	304317.0684	321.6678	305.797
18	GMT 017	1305210.007	304610.2349	321.7178	305.831
19	GMT 018	1305196.065	304426.5327	322.7238	306.809
20	GMT 019	1304779.795	303517.2379	330.7478	314.451
21	GMT 020	1305182.445	303609.2585	325.6728	309.76
22	GMT 021	1305298.922	303213.4267	325.9688	310.075
23	GMT 022	1305428.38	302647.6772	331.4428	315.58
24	GMT 023	1305618.345	302811.3058	326.8368	310.925
25	GMT 024	1305730.036	302267.2404	331.2898	315.299
26	GMT 025	1306046.451	302816.7172	323.7868	307.958
27	GMT 026	1306167.177	302599.6514	324.0828	308.26
28	GMT 027	1306166.226	303203.0634	322.4758	306.641
29	GMT 028	1306634.059	303018.7848	320.8018	304.94
30	GMT 029	1306827.646	303186.7234	319.7148	303.784
31	GMT 030	1307086.880	303388.3263	318.9648	303.053

The coordinates easting and northing were used to compute Y_o and x_o using (Equation 9) as the mean as well as the centroid of the positions given the reference coordinates as:

Y o□□□□1305749.762

X o□□□□303830.8077

Computed Geoidal Height (N) from Observed Ellipsoidal (h) and Orthometric (H) Heights

The geoidal height (N) computed from the observed ellipsoidal and orthometric heights (H) are presented in Table 2 below.

Table 2: Results of Geoidal Height from GPS & Geodetic Levelling

STATIONS	Ellipsoidal Height (h)m	Orthometric Height (H)m	Geoid height(N)m
BSC_10	335.1683	319.27	15.8983
GMT 001	331.9368	316.023	15.9138
GMT 002	322.2108	306.305	15.9058
GMT 003	321.4848	305.61	15.8748
GMT 004	319.3148	303.45	15.8648

GMT 005	318.6078	302.77	15.8378
GMT 006	319.0708	302.248	16.8228
GMT 007	318.0498	302.235	15.8148
GMT 008	318.6718	302.765	15.9068
GMT 009	320.8228	304.816	15.9368
GMT 010	320.0298	304.29	15.7398
GMT 011	321.6768	305.297	16.3798
GMT 012	323.3268	307.393	15.9338
GMT 013	320.5218	304.834	15.6878
GMT 014	320.0548	304.19	15.8648
GMT 015	320.3478	304.452	15.8958
GMT 016	321.6678	305.797	15.8708
GMT 017	321.7178	305.831	15.8868
GMT 018	322.7238	306.809	15.9148
GMT 019	330.7478	314.451	16.2968
GMT 020	325.6728	309.76	15.9128
GMT 021	325.9688	310.075	15.8938
GMT 022	331.4428	315.58	15.8628
GMT 023	326.8368	310.925	15.9118
GMT 024	331.2898	315.299	15.9908
GMT 025	323.7868	307.958	15.8288
GMT 026	324.0828	308.26	15.8228
GMT 027	322.4758	306.641	15.8348
GMT 028	320.8018	304.94	15.8618
GMT 029	319.7148	303.784	15.9308
GMT 030	318.9648	303.053	15.9118

Figure 5 presents the comparisons of ellipsoidal, orthometric and geoidal height for the 31 points covered within the study area.

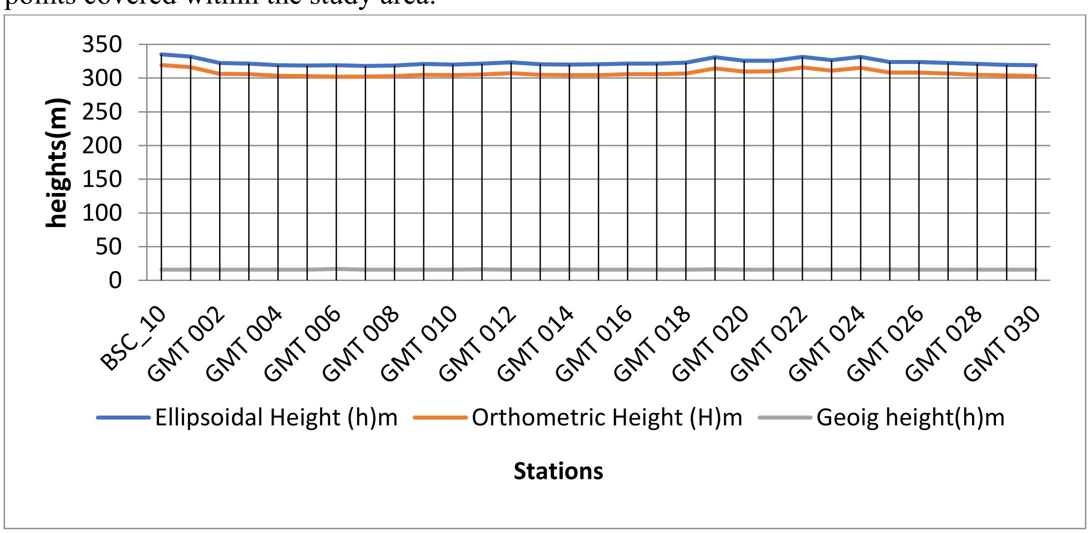


Figure 5: Comparison of Ellipsoidal,Orthometric Height, and the Geoidal height.

Numerical Application for Determination of Local Geoidal (N) using Polynomial Interpolation Model

Deducting the values of Easing (x) and Northing (y) for each equation using equation (9) for the computation of coefficient matrix, A, elements as presented in Table 3.

Table 3: Computation of Coefficient Matrix, Elements

a	X	y	x2	y2	Xy	x2y	xy2	x3	y3	x3y3
1	263.2547387	1733.60067	69303.05745	3005371.276	456378.5909	120143826.7	791178230	18244358.3	5210113652	95055180141123200.00
1	181.4685387	1387.99937	32930.83054	1926542.246	251878.217	45707971.97	349606806	5975909.7	2674039419	15979818092839600.00
1	936.6690387	825.764068	877348.8881	681886.2955	773467.6355	724483186.6	638701781	821785540	563077201.1	462728701520174000.00
1	1032.214539	608.911968	1065466.855	370773.7844	628527.786	648775518.9	382718091	1099790378	225768594.6	248298128048664000.00
1	984.6171387	130.057932	969470.9098	16915.06575	128057.2692	126087382	16654863.6	954557673	2199938.477	2099968153684740.00
1	864.6171387	427.976732	747562.7965	183164.0834	370036.0177	319939482.8	158366806	646355606	78389965.88	50667793914095300.00
1	543.2629387	768.249132	295134.6206	590206.7293	417361.2813	226736916.2	320637442	160335701	453425807.6	72700344847596600.00
1	612.6853387	1213.61153	375383.3243	1472852.951	743561.9925	455569531.2	902395409	229991859	1787471326	411103853423687000.00
1	140.4920387	1081.80783	19738.01294	1170308.185	151985.3878	21352736.98	164418983	2773033.68	1266048561	3510795296505930.00
1	125.3838387	507.003632	15721.10701	257052.6832	63570.06165	7970658.356	32230252.2	1971172.75	130326644.1	256896328735205.00
1	260.9211387	152.307623	68079.84062	23197.61208	39740.27848	10369078.71	6052747.36	17763469.5	3533173.161	62761413807257.30
1	133.020587	120.740468	17694.47657	0.548292477	98.4974537	13102.18911	72.9341871	2353729.66	0.405992892	955597.51
1	292.0134613	16.8624323	85271.86158	284.3416217	4924.05721	1437890.99	83031.5812	24900531.5	4794.691335	119390362387.93
1	413.6785387	30.3063677	171129.9334	918.4759256	12537.09392	5186326.692	379953.779	70792780.8	27835.66916	1970564424492.85
1	772.5155387	214.896768	596780.2575	221.9136891	11507.98456	8890096.888	171431.773	461022022	3305.796685	1524045072408.33
1	856.0035387	274.857168	732742.0583	75546.46264	235278.7082	201399406.8	64668039.4	627229795	20764486.75	13024104763908600.00
1	486.2607387	296.572868	236449.506	87955.46586	144211.7417	70124508.06	42769289.8	114976112	26085204.74	2999175407324830.00
1	779.4272387	539.754668	607506.8204	291335.1013	420699.4902	327904642	227074514	473507364	157249480.8	74458787068200500.00
1	595.7250387	553.696368	354888.3217	306579.6676	329850.7901	196500374.7	182637184	211415859	169752048.4	35888275155515700.00
1	131.5697613	696.966368	17310.60209	485762.1177	91699.69863	12064907.46	63911605.9	2277551.79	338559858.7	771087610524421.00
1	221.5491613	567.316668	49084.03087	321848.2015	125688.5319	27846188.83	71305199.1	10874525.9	182589849.2	1985578038764550.00
1	617.3809613	450.839768	381159.2514	203256.4961	278339.8892	171841748.3	125486691	235320465	91636111.5	21563852371802400.00
1	183.130461	321.381268	1399797.688	103285.9192	380235.9674	449868755.4	122200717	1656143284	33194159.66	54974284571344600.00
1	1019.501861	131.416368	1039384.045	17270.2617	133979.2314	136592075.8	17607063.9	1059653968	2269595.062	2404985412382010.00
1	163.567261	219.725268	26754.24887	389.0861874	3226.408017	527734.7222	63641.762	4376119.21	7674.829221	33585967570.39
1	1014.090461	296.689532	1028379.463	88024.67858	300870.0246	305109421.9	89264986.9	1042869804	26116000.72	27235588544732900.00
1	1231.156261	417.415332	1515745.739	174235.5596	513903.4998	632695511.3	214511200	1866119857	72728594.03	135720273456976000.00
1	627.7442661	416.464632	394062.8637	173442.79	261433.285	164113245.6	108877717	247370703	72232787.74	17868275495313900.00
1	412.0228613	184.297432	659381.1273	781981.9488	718069.7312	583089037.8	634987220	535432550	691504629.4	370254086820988000.00
1	644.0842613	1077.88473	414844.5357	1161835.495	694248.5914	447154591.1	748319957	267194836	1252324742	334614704347140000.00
1	442.4813613	1337.11793	195789.7551	1787884.364	591649.7628	261793992.4	791105507	86633317.4	2390612244	207106669190339000.00

Geoidal Undulation (N) Determined from the Coordinates

Computation of the model parameters, X using equation (6) Using coefficient matrix, A as well as the computed coefficient matrix, A elements given in table 2 and the observation matrix, L, the model parameters are computed as:

$$X=\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \end{pmatrix} = \begin{pmatrix} 13.4245267504612000000000000000 \\ 0.0100302495182447000000000000 \\ 0.0028957751121012600000000000 \\ -0.0000258270882720936000000000 \\ -0.0000049942502909263600000000 \\ 0.0000292045648963013000000000 \\ -0.0000000243651788725725000000 \\ -0.00000001654049421038380000000 \\ 0.0000000176875202445150000000 \\ 0.0000000023677414930051000000 \\ 0.0000000000000000140884932994 \end{pmatrix}$$

The computed parameters are Substitute into the interpolation model. The computed parameters, were substituted into the geometric geoid model as;

$$N = 13.424526750461200 + 0.010030249518244 70X + 0.002895775112101260Y + -0.000025827088272093600X^2 + -0.000004994250290926360Y^2 + 0.000029204564896301 3000XY + -0.000000243651788725 7250X^2Y + -0.000000165404942103838 00XY^2 + 0.0000000176875202445 1500X^3 + 0.000000002367741493005 1000Y^3 + 0.000000000000000001408849329 94Y^3X^3$$

The Microsoft Excel 2010 was used to program the polynomial surface models for interpolation of geoid undulation.

Table 4: Computed geoid undulation from local polynomial

STATIONS	LOCAL (N)	STATIONS	LOCAL (N)
set_base	15.6	GMT 016	16.54786
GMT 001	15.9137	GMT 017	15.99773
GMT 002	15.9058	GMT 018	16.77775
GMT 003	15.8748	GMT 019	16.06909
GMT 004	15.8647	GMT 020	16.87987
GMT 005	15.8378	GMT 021	16.61243
GMT 006	16.8228	GMT 022	16.14348
GMT 007	15.8148	GMT 023	16.17581
GMT 008	15.9068	GMT 024	15.64826
GMT 009	15.9368	GMT 025	16.22316
GMT 010	15.7398	GMT 026	16.71323
GMT 011	15.00937	GMT 027	17.56254
GMT 012	14.74642	GMT 028	15.48514
GMT 013	14.72012	GMT 029	15.8975
GMT 014	14.07299	GMT 030	15.7434
GMT 015	15.72572		

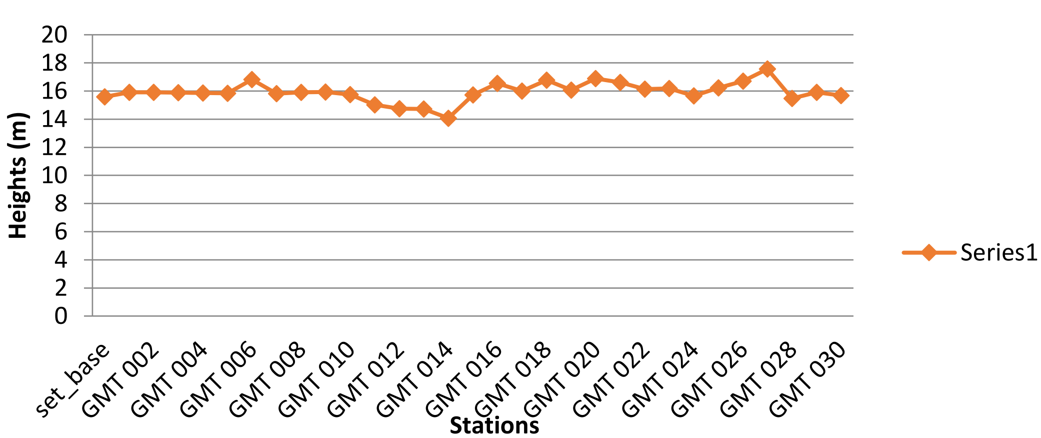


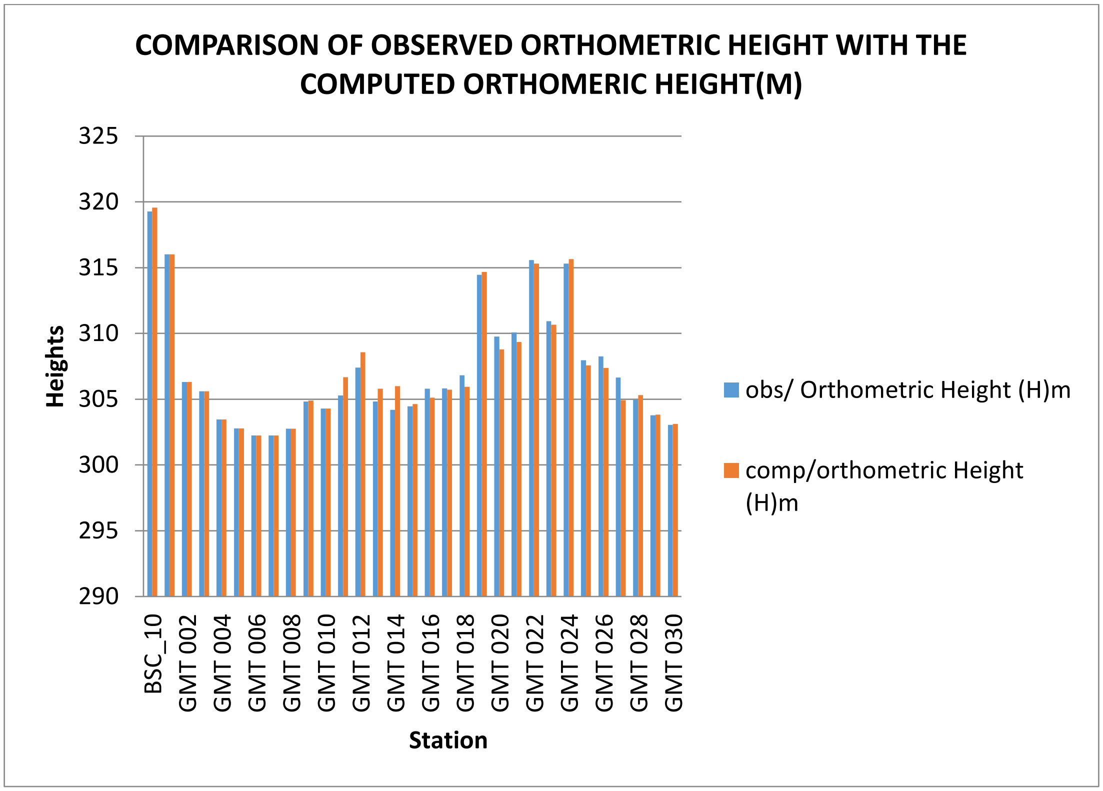
figure 6: Computed Local Geoid Heights Profile

Orthometric Height Determined from Ellipsoidal Height

The orthometric heights were determined from Computed geoid undulation of local polynomial model using equation 2.7 and presented in Tables 5 below.

Table 5: Orthometric Height Determined from Ellipsoidal Height using local geoid model

STATIONS	Ellipsoidal Height (h)m	Local(N)	Comp(H)
BSC_10	335.1683	15.6001	319.568
GMT 001	331.9368	15.9137	316.023
GMT 002	322.2108	15.9058	306.305
GMT 003	321.4848	15.8748	305.610
GMT 004	319.3148	15.8647	303.450
GMT 005	318.6078	15.8378	302.770
GMT 006	319.0708	16.8228	302.248
GMT 007	318.0498	15.8148	302.235
GMT 008	318.6718	15.9068	302.765
GMT 009	320.8228	15.9368	304.886
GMT 010	320.0298	15.7398	304.290
GMT 011	321.6768	15.00937	306.667
GMT 012	323.3268	14.74642	308.580
GMT 013	320.5218	14.72012	305.801
GMT 014	320.0548	14.07299	305.981
GMT 015	320.3478	15.72572	304.622
GMT 016	321.6678	16.54786	305.119
GMT 017	321.7178	15.99773	305.720
GMT 018	322.7238	16.77775	305.946
GMT 019	330.7478	16.06909	314.678
GMT 020	325.6728	16.87987	308.792
GMT 021	325.9688	16.61243	309.356
GMT 022	331.4428	16.14348	315.299
GMT 023	326.8368	16.17581	310.660
GMT 024	331.2898	15.64826	315.641
GMT 025	323.7868	16.22316	307.563
GMT 026	324.0828	16.71323	307.369
GMT 027	322.4758	17.56254	304.913
GMT 028	320.8018	15.48514	305.316
GMT 029	319.7148	15.8975	303.817
GMT 030	318.9648	15.8434	303.121
	10019.19	**494.0697**	**9525.122**



Observed Geoidal Heights Compared with Computed local Geoidal

Figure 7: Comparison of Observed Orthometric Height, with the Computed Orthomeric Height model

The Observed Geoidal Heights from Table 2 above are compared with Computed Geoidal Height.

Table 6: Comparison of known geoid undulation with computed local polynomial model

STATIONS	KNOW (N)	LOCAL (N)
set_base	15.598	15.600
GMT 001	15.914	15.914
GMT 002	15.906	15.906
GMT 003	15.875	15.875
GMT 004	15.865	15.965
GMT 005	15.838	15.838
GMT 006	16.823	16.823
GMT 007	15.815	15.815
GMT 008	15.907	15.907
GMT 009	15.937	15.937
GMT 010	15.74	15.740
GMT 011	15.78	15.348

GMT 012	15.934	14.746
GMT 013	15.688	14.720
GMT 014	15.865	14.073
GMT 015	15.896	15.726
GMT 016	15.871	16.548
GMT 017	15.887	15.998
GMT 018	15.915	16.778
GMT 019	16.297	16.069
GMT 020	15.913	16.880
GMT 021	15.894	16.612
GMT 022	15.863	17.822
GMT 023	15.912	16.176
GMT 024	15.991	14.587
GMT 025	15.829	16.223
GMT 026	15.823	18.100
GMT 027	15.835	17.563
GMT 028	15.862	13.219
GMT 029	15.931	15.897
GMT 030	15.912	15.834
sum	**493.11**	**494.078**

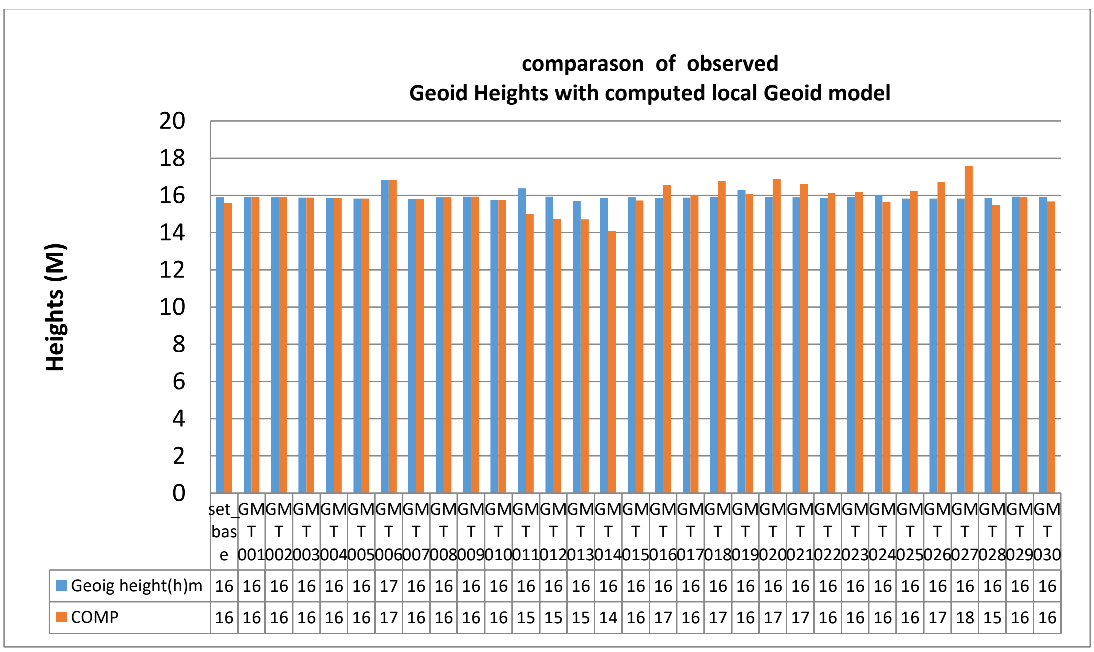


Figure 8: Comparison of Known Geoidal Height and Computed Geoidal Height

Several statistical calculations for sufficient performance of the model were made. These statistics revealed the difference between the known and local geoidal undulations (N_{KNOWN} and N_{local}) as; $\Delta N_{LOCAL} = 493.110 - 494.078 = -0.9674$.

In addition, the accuracy of the geometric geoid model was verified by finding the difference between the calculated and measured values. This results to RMSE of ± 0.173741219448502 (≈0.174). Suffer software was used to plot the 3D surface and contours using a contour interval of 0.2m as shown in Figures 9 and 10 below.

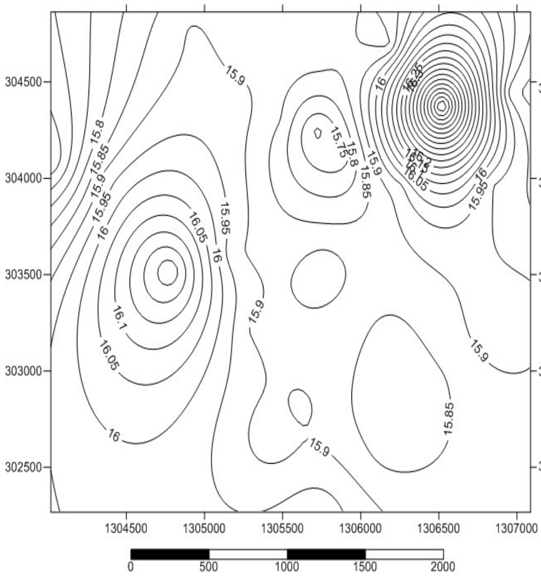


Figure 9a contour map of known geoidal model

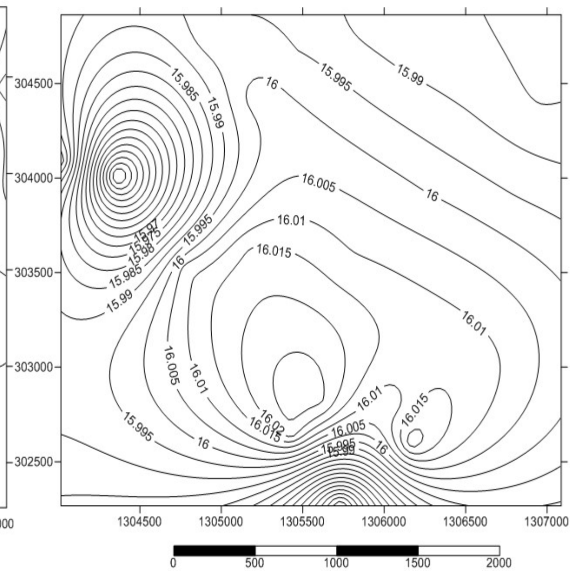


Figure 9b contour map of Local Geoidal model Heights

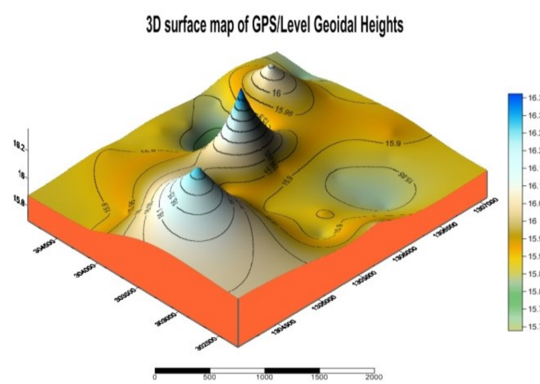


Figure 10a 3D surface map of GPS/Level Geoidal model Heights

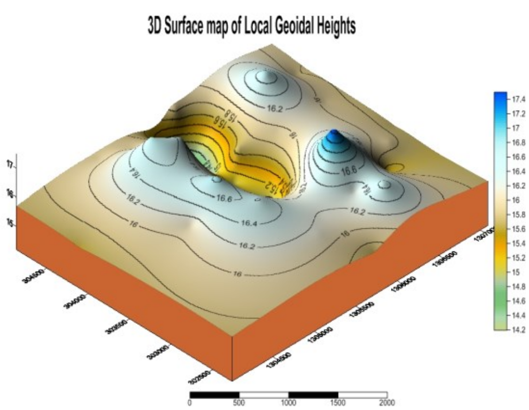


Figure 10b 3D surface map for Local Geoidal model Heights

Discussion

This study, polynomial interpolation model was utilized to determined local geoid model of 31 points and the adjusted geoid model (N_{adjust}) depending on the GPS/level geoid heights related to M.S.L, the difference between GPS/level geoid heights and the one acquired local geoid model, from the GPS-RTK method. The mathematical statistics of the model as a result, of the local geoid height's standard deviation of 0.171meters, and 0.50134meters respectively, as well as The RMSE value is 0.174 and 0.509 centimeters respectively, the correction mean values were 0.026 and -5876m. Suffer software was used to plots 3D surface and contour showed in Figures 9a and 9b for general mapping the contour intervals at 0.2m.

Conclusion

A geoid model is particularly significant to transform the GPS dependent ellipsoidal heights into orthometric heights which are commonly used in geomatics, mapping, surveying applications and GIS. Up to date, there is no national and accurate geoid model in Maidugiri, Borno State, Nigeria. This study has explored the GPS/leveling geoid modeling method as well as amplifying its accuracy by using the polynomial technique. A small area within the University of Maiduguri was utilized to determined local geoid for the Borno State. A complete geoid model for the State is absent.

Thus, a standard geoid model is important for the entirety of the region. Based on the results obtained, the study recommends that advanced techniques in the field of surveying should be using, sure as drone and unmanned aerial vehicle (UAV) with low altitude to determine the geoid model for the entire region. Therefore, further research and development must be undertaken.

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