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Empirical Modelling of Mechanical Properties of Concrete Using Bida Natural Gravel as Coarse Aggregate

Bala Alhaji Abbas
Department of Civil Engineering, Federal University of Technology, Minna, Nigeria.
Corresponding Author: bala.alhaji@futminna.edu.ng
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Abstract

Strength performance remains the most important property of structural concrete, from engineering point of view. Compressive, flexural, tensile strengths and elastic modulus of concrete are of great importance in structural engineering. The determination of compressive strength of concrete is easier to carry out in the field when compared to flexural tensile strengths and elastic modulus. It is therefore, customary to determine the compressive strength and correlate it to other strength properties. In this study, empirical relationships have been developed to relate the compressive strength to the flexural, splitting tensile strengths and elastic modulus of concrete using Bida gravel as coarse aggregate. Extensive experimental work was conducted using 400 specimens for compressive strength, 400 for flexural strength, 400 for splitting tensile strength and 400 for elastic modulus (1,600 in all). Central composite design was used for the factor setting with the following range of values; W/C= 0.4, 0.5, 0.6, CA/TA= 0.55, 0.615, 0.68, TA/C= 3.0, 4.5 and 6.0. The specimens were produced and cured for 7, 14, 21 and 28 days. The compressive, flexural and splitting tensile strengths of concrete samples from these mixes were determined at 7, 14, 21 and 28 days of age. From the strength data obtained at 28 days, regression equations were developed that relate the strength properties with the aid of regression analysis tool in Microsoft Excel. The empirical models developed to predict the flexural, splitting tensile strengths and elastic modulus of concrete from the compressive strength recorded R^2 values of 0.99, 0.99 and 1, P-values of 2.41×10^{-9}, 1.6×10^{-9}, and 6.7×10^{-10} standard errors of 0.684, 0.19 and1.42respectively. Furthermore, residuals from the values of predicted strength properties show that there is very slight deviation between the experimental and predicted values. It was concluded that the empirical equations developed are significant, have high predictive capabilities and can be used in predicting the flexural, splitting tensile strengths and elastic modulus of concrete.

Keywords: Flexural Strength, Splitting Tensile Strengths, Elastic Modulus, Bida Gravel, Response Surface Methodology, Central Composite Design, Regression Modelling.

Introduction

Concrete is an artificial stone produced by careful selection and addition of constituent materials which include cement, aggregate [fine and coarse] and water with or without admixture in a right proportion. When the constituents are added together, cement and water react together and bind the aggregate together gradually and consequently concrete gain strength with age (Abbas *et al*., 2024).

Compressive strength of concrete is the most common measure for judging the quality of concrete and thus strength of concrete is usually determined through the use of cubes or cylinder mould which constitute the shape of specimen. In addition, the type of aggregate used have some effect on the compressive strength of concrete. Tensile strength measures a materials ability to withstand tensile forces while flexural strength measures a material ability to withstand bending stress. Understanding the similarities and differences between these two properties is crucial in selecting the right material for a specific application (Neville and Brooks, 2002).

Compressive strength, flexural strength and tensile strength are important properties of concrete that one utilized for design of structure. Tensile strength of concrete can be estimated by three methods which include modulus of rupture test, direct tension and split cylinder test. Study has shown that the split cylinder test is simple. However, this is not always easy from an experimental point of view to, thus to reduce the energy and time consumption in direct measurement of the splitting tensile strength, researchers have tried to predict the splitting tensile strength and flexural strength using theoretical and empirical approaches based on compressive strength (Marin-Uribe and Navarro-Gaete, 2021).

Flexural strength is the ability of a material to resist deformation under bending forces, representing the maximum stress it can withstand before fracturing. It is also known as bending strength or modulus of rapture and determined through a benching test such as four-point test, this property is a critical factor for material in structural applications (Shetty, 2005).

Tensile strength is maximum amount of stress a structure can withstand when being pulled or stretch before it breaks or deforms. It is a key mechanical property that is critical for engineers in determining the material ability to resist tensile force in application like construction and manufacturing of structures.

Research Significance

The important of empirical relationship between compressive strength, flexural, tensile strength and elastic modulus lies in its ability to allow engineers to predict the material flexural strength, tensile strength and elastic modulus using compressive strength which is easier and more convenient to conduct. This relationship is crucial for structural design as it helps in estimating the

flexural, splitting tensile strength and elastic modulus of a structural member. In other words, Compressive strength is the primary and easiest property to test in material like concrete. The empirical relationship allows for the estimation of tensile and flexural strength without the need for more complex and time-consuming test. Understanding the tensile strength is vital for the serviceability of a structure as it indicate when cracking will occur. By knowing the tensile strength this can be used to estimate the load under which crack will initiate and develop. In addition, most of the research were published to determine the concrete properties using few mixes (that is with very few samples). However, this research uses 1,600 (one thousand six hundred sample).

Pebbles (BNA) has gain enormous attention in the usage as a coarse aggregate, but less research has been done on it compare to conventional gravel, this aggregate is by far cheaper than the crushed granite. No empirical equation has been developed to predicted the flexural, splitting tensile strength and elastic modulus of the concrete made using this aggregate with respect to compressive strength using response surface methodology for the mixed proportion (Abbas *et al*., 2024).

It is against this background that this research is aimed at developing an empirical relationship for compressive strength and flexural strength, compressive strength and splitting tensile strength, and compressive strength and elastic modulus of concrete using pebbles (Bida natural stones) as coarse aggregate.

Experimental Program

Pebble use in this experimental work was obtained from Bida environs (Alhaji *et al*., 2024). The aggregate is between 20mm-5mm in size reddish brown, smooth and round in shape. The physical properties of the aggregate are given in Table 2.0. The coarse aggregate fall in zone 31. Table 3.0 shows the Concrete Components Required for $1m^3$ Concrete of Different Mix Proportions for design mixes developed for the cubes of $150mm^3$, cylinder of diameter 150mm x 300mm and prism of $100x100x500mm^3$ for compressive strength, splitting tensile strength, flexural strength and elastic modulus respectively).

Previous Research Literature Review

Efforts were made by several researchers to develop empirical equation that relate compressive strength, splitting tensile strength and flexural strength of a concrete as follow:

Equation $0.2784f_{cu}$ and $0.0849f_{cu}$ where develop as empirical relation between compressive strength and flexural strength, and compressive strength and splitting tensile strength respectively by Abubakar *et al*. (2025).

According to Marin-Uribe and Navarro-Gaete (2021) who developed empirical relationships between compressive and flexural strengths of concrete using reclaimed asphalt. Reclaimed asphalt was incorporated into the concrete mix at 0, 20, 50 and 100%. The study developed linear, logarithmic and power relationships that relate concrete cube compressive strength (f_{cu}) and concrete beam flexural strength (f_s). as $f_s = 0.00524f_{cu} + 1.3894$, $f_s = 0.9948 \ln f_{cu} - 0.4438$ and $fs = 0.6647(fcu)0.4417$ respectively

Research was conducted by Franklin and Kangootui (2020) using Kgale aggregates with botchem as a binding agent who developed empirical relationships relating compressive strength and flexural strength, and compressive strength and splitting tensile strength as $f_r = 0.234(f_c')^{2/3}$ and $0.533(f_c')^{0.5}$

Juki *et al*. (2013) developed empirical relationship between compressive, flexural and splitting tensile strength of concrete using laboratory results from properties of concrete, containing granulated waste Polyethylene Terephthalate (PET) bottles as fine aggregate. Fine aggregate was replaced at 25, 50 and 75%. Equations $f_{fs} = 0.466(f_c)^{0.703}$ and $f_t = 0.634(f_c)^{0.5}$ shows the relationship between flexural and compressive strength, and splitting tensile and compressive strength respectively.

Chekravarty *et al*. (2022) conducted a research work on evaluation of empirical relation between compressive, Flexural, Split Tensile Strength of Concrete by Partially Replacing Nano Silica/ the research concluded a good compliance between experimental and analytical results was observed. Equation ft = 0.059f $ck^{1.087}$ was developed. where ft = Split tensile strength and fck = Compressive strength. It has R^2=0.9545.

Sheikha and Qadir (2017) conducted research to develop a Multi Variable Empirical Formula for SFRC Compressive, Tensile, and Shear Strength, of a concrete and arrived at the following. Three equations were developed for compressive strength, and two were developed for tensile strength and shear strength each**.**

Compressive strength

i) σ Compressive = 100.2257+ 2.913508 (Vt) – 91.0231 (w/c) – 0.05945 (C)

ii) σ Compressive = 89.7373202+ 3.052725 (Vt) – 107.3881976 (w/c) – 0.094657534 (P)

iii) σ Compressive = 45.04102217+ 2.423448276 (Vt) – 0.740973892 (P)

Tensile strength

i) σ Tensile = 0.53325 (Vt) - 5.990202055+ 0.026481267 (C)– 0.0085273947(P)

ii) σ Tensile = 5.49576127+ 2.31024379 (Vt) – 0.036534039* (P)

Shear strength

i) σ Shear = 4.02484485 (Vt) - 3.464421798 - 0.112666667(P)+ 0.015061828*(γ),

ii) σ Shear = 3.386443374+ 0.616343137 (Vt) + 0.002936231 (w/c)

Where;

σ = Compressive of SFRC

Vt = percentage of Steel fiber

W/c = Water cement ratio

C = Cement content

ρ = Aspect ratio

γ = Aggregate content

Some empirical relationship to predict the mechanical properties of concrete through compressive strength were establish by ACI 318-99, CEB-FIP and EURO CODE as follows.

Table 1: Empirical relationship between compressive strength and other mechanical properties

SOURCE	FLEXURAL STRENGTH	SPLITTING TENSILE STRENGTH	ELASTIC MODULUS
ACI 318 -99	$0.62\sqrt{f_{cu}}$	$0.56\sqrt{f_{cu}}$	$4700\sqrt{f_{cu}}$
CEB-FIP	$0.6\sqrt{f_{cu}}$	$0.3fcu^{2/3}$	-
EURO CODE 2	$0.30xfck^{2/3}$	-	$22000(fcu/10)^{0.3}$
IS 456-2000	$0.7\sqrt{f_{cu}}$	-	$5000\sqrt{f_{cu}}$

MATERIALS AND METHOD

Materials

The materials used for this research work are as follows;

Cement: Ordinary Portland cement used for this research was purchased at cement deport located at Kpakungu, Minna, Niger state. It was produced in accordance with BS EN 197-1(2000): Part 1 and classified as CEM 1 of the standard.

Sand: The sand was obtained from Gidan Mangoro, Minna, Niger state, the sand conforms to the standard requirement of BS EN 12620 (2008).

Water: Clean water fit for drinking sourced from Civil Engineering laboratory, Federal University of Technology Minna was used for this research work, the water was clean as recommended by BS EN1008 (2002).

Gravel: The coarse aggregate (plate I) used for this work was taken from Bida, Niger state in the middle belt region of Nigeria located about 250Km North-East of Bida inland from the Federal capital city of Abuja, Nigeria. The deposits aggregates lie in zone 31, which fall within the latitude N 90 55′E and longitude N 50 52′ E (Abbas *et al*., 2024). The Bida natural deposits aggregates in character occur in middle Niger basin of Nigeria in several million metric tons. This aggregate is characterized with round shape and smooth surface.



Plate I: Bida natural coarse aggregate

Methods

Laboratory Tests Conducted on the Aggregate Include:

Moisture content, specific gravity, sieve analysis, aggregate impart value (AIV), aggregate crushing value (ACV), bulk density (compacted and uncompacted), water absorption and silt test were conducted in accordance with BS EN 12620 (2008).

Laboratory Test Conducted on the Fresh and Hardened Concrete Include:

Slump test, compacting factor test (compacted and uncompacted) were conducted on the fresh concrete in accordance with BS EN 12350-1 (2009) and compressive strength, splitting tensile strength, flexural strength and elastic modulus test respectively were conducted on the hardened concrete in accordance. BS EN 12390 (2002)

The Concrete Specimen Produced Manually and Tested Were:

Cube (150 x 150 x150 mm^3), prism (100 x100 x 500^3), cylinder (150 x 300 mm^2) and also cylinder (150 x 300 mm^2) for compressive strength, flexural strength, splitting tensile strength and elastic modulus test respectively in accordance to specifications BS 12390 (2002).

Factor Setting

Central Composite Design (CCD) method was used for the factor settings. Factor setting involves changing one or more factor at a time to study the effect of the variable on the response, (Carley, 2004). This operation was randomized by the software such that the experimental result from the laboratory yielded the best proportion. In this study, four (4) concrete components of water, cement, coarse aggregate and fine aggregate were used with three (3) corresponding independent variables/factors of water-cement ratio (W/C), coarse aggregate-total aggregate (CA/TA) and total aggregate-cement (TA/C). The independent variable ware represented by x_1, x_2 and x_3 respectively. The ranges of the independent variables chosen are as stated below:

W/C = 0.4, 0.5 and 0.6

CA/TA = 0.55, 0.615 and 0.68

TA/C = 3.0, 4.5 and 6.0

Absolute volume method was used for mix proportioning.

Production of Concrete for Compressive Strength, Flexural Strength, Splitting Tensile Strength and Elastic Modulus Test

The concrete constituents were mixed thoroughly (manually) using mix proportion until the required homogeneity was achieved; the mould was lubricated with engine oil in order to reduce friction such that the product can be removed easly. The moulds were filled with fresh concrete in three layers and each layer was tamped 25 times to produce cubes of 150x150x150mm^3, prism of 100x100x500mm^3, cylinder of 150 x 300mm^2.

Number of Samples Produced for the Experiment

- Curing days = 7, 14, 21 and 28 days = 4
- Number of trials per each property = 5
- Number of sample point generated by minitab14 package = 20
- Number of hardened properties, compressive strengths, flexural strengths, splitting tensile strengths and modulus of elasticity = 4
- Total number of samples used = 4x5x20x4 = 1,600
- Four hundred (400) per each hardened property (that is compressive strengths = 400, flexural strengths = 400, splitting tensile strengths = 400 and modulus of elasticity = 400)

Crushing and Curing of the Concrete Sample

Curing

The total immersion method of curing was adopted, the hardened concrete samples were demoulded after 24 hours and kept in the water for 7, 14, 21 and 28 days respectively, the mechanical properties that were determine include compressive strength, splitting tensile strength, flexural strength and elastic modulus.

Crushing

Compressive Strength Test

Four hundred (400) specimens were crushed at the age of 7, 14, 21, and 28days respectively using an electrically operated Seidner compressive machine at Civil Engineering laboratory, Federal University of technology, Minna (destructive test) in accordance wit h BS EN 12390 (2002). The weight of each cube was alwa ys taken before the compressive strength test. Neville (2003).

Flexural Strength

Four hundred (400) specimens were crushed at the age of 7, 14, 21, and 28days respectively at Civil Engineering laboratory, Federal University of technology, Minna (destructive test) in accordance wit h BS EN 12390 - 6 (2009). The weight of each cube was always taken before the compressive strength test.

Splitting Tensile Strength

Four hundred (400) specimens were crushed at the age of 7, 14, 21, and 28days respectively at Civil Engineering laboratory, Federal University of technology, Minna (destructive test) in accordance wit h BS EN 12390 (2002). The weight of each cube was alwa ys taken before the compressive strength test.

Elastic Modulus Test

Four hundred (400) specimens were crushed at the age of 7, 14, 21, and 28days respectively at Civil Engineering laboratory, Federal University of Technology, Minna (destructive test) in

accordance with BS EN 12390 (2002). The weight of each cube was always taken before the compressive strength test.

Table 2: Summary of preliminary test

Test	Bida natural stones	Sand
Specific gravity	2.65	2.57
Bulk density (kg/m^3) (compacted)	1781.09	1618.30
Bulk density (kg/m^3) (uncompacted)	1618.56	1467.30
Moisture content	-	7.68
Water absorption	0.55	0.76
ACV	38.09%	-
AIV	40.56%	-

Table 3: Concrete Components Required for 1m^3 Concrete of Different Mix Proportions for (Cube, prism and cylinder).

S/No	Water (kg/m^3)	Cement (kg/m^3)	Fine aggregates (kg/m^3)	Coarse aggregates (kg/m^3)
1	193.09	386.19	834.17	903.68
2	193.55	387.09	661.93	1079.99
3	193.95	387.91	506.22	1239.37
4	147.51	409.75	700.67	1143.2
5	193.55	387.09	661.93	1079.99
6	146.74	293.47	738.26	1204.54
7	321.96	536.6	574.70	702.42
8	193.55	387.09	661.93	1079.99
9	234.76	366.81	627.24	1023.39
10	193.55	387.09	661.93	1079.99
11	210.41	526.02	710.12	867.93
12	183.25	305.42	824.65	1007.90
13	130.74	326.84	627.54	1333.52
14	193.55	387.09	661.93	1079.99
15	284.21	568.41	514.07	838.748
16	183.84	306.41	588.30	1250.13
17	130.29	325.73	879.47	1074.90
18	285.73	476.21	457.16	971.47
19	210.99	527.47	506.37	1076.04
20	193.55	387.09	661.93	1079.99

Table 4: Measured Responses at 28 days Curing

Mix Order	W/C X1	CA/TA X2	TA/C X3	Compressive Strengths (Y_1) N/mm^2	Flexural Strengths (Y_2) N/mm^2	Splitting Tensile Strengths(Y_3) N/mm^2	Elastic Modulus (Y_4) kN/mm^2	Slump (Y_5) Mm
1	0.5	0.52	4.5	27.00	7.25	2.29	20.06	0
2	0.5	0.62	4.5	25.50	6.9	2.15	16.77	35
3	0.5	0.71	4.5	27.70	7.7	2.26	20.06	60
4	0.36	0.62	4.5	24.00	6.35	1.93	15.16	0
5	0.5	0.62	4.5	25.70	6.95	2.26	17.32	50
6	0.5	0.62	6.62	19.15	3.2	1.08	12.94	0
7	0.6	0.55	2.38	23.00	5.9	1.92	15.08	245
8	0.5	0.62	4.5	25.33	6.8	2.14	20.04	40
9	0.64	0.62	4.5	21.46	5.35	1.78	15.07	190
10	0.5	0.62	4.5	25.38	6.8	2.14	15.80	35
11	0.4	0.55	3.0	30.13	7.55	2.53	22.43	60
12	0.6	0.55	6.0	19.48	3.7	1.55	14.01	20
13	0.4	0.68	6.0	12.96	2.2	0.75	10.14	0
14	0.5	0.62	4.5	25.90	7.0	2.29	20.06	40
15	0.5	0.62	2.38	29.16	5.95	1.93	18.91	210
16	0.6	0.68	6.0	22.50	5.4	1.78	15.09	35
17	0.4	0.55	6.0	7.91	1.6	0.72	7.21	0
18	0.6	0.68	3.0	21.59	5.25	1.68	14.13	210
19	0.4	0.68	3.0	33.42	7.9	2.62	25.09	70
20	0.5	0.62	4.5	26.00	7.15	2.29	21.05	35

Developing Empirical Models that Relates the Mechanical Properties

Relationships between compressive and flexural strengths, compressive and splitting tensile strength, and compressive and elastic modulus were developed. The relationships between these mechanical properties of concrete in this study were developed using the regression analysis tool pack in excel. For each pair of relationship, three forms of empirical equations were developed. These equations are in the form f_t =kfcu^n the values for n were pre-assigned as 1, ½ and 2/3 to ensure adequate comparison to existing empirical equations while *K*, the coefficients of compressive strength were generated using excel regression tool pack.

RESULTS AND DISCUSSION

Characterization of Aggregate

The result of physical properties of the aggregate presented in Table 2. Shows that the aggregate can be conveniently use for construction work (concrete) without much need for mix proportioning adjustment (Naville, 1995). From the Table 2, the average result of aggregates crushing value (ACV) for the Bida natural aggregates is 38.0%. Usually, a material that has undergone a complete process of physical weathering offers a less resistance to crushing value. BS-EN: 13043 (2002) identified that, aggregates crushing value (ACV) below 10% is regarded as very strong aggregates while those above 35% are regarded as weak

aggregates and should not be used as wearing surface concrete but can be used for normal concrete work. Hence, this aggregate is not suitable for wearing surface concrete, since the ACV is above 35%, also the result of average Aggregates Impact Value (AIV) for Bida Natural Coarse Aggregates is 40.56%, BS EN 13043 (2002) identified that, AIV of material for concrete pavement must not exceed 30% and must be less than 45% for concrete wearing surface. Therefore, this aggregate with AIV of 40.56% which is greater than 30% but less than 45% is suitable for concrete wearing surface and but can also be used for normal concrete work.

Properties of Hardened Concrete

The summary of experimental results of concrete crushed at 28days curing for compressive strength, flexural strength, splitting tensile strength and elastic modulus are respectively presented in Table 4.

The strength increases as the curing age increases at all cases tested, this is primarily due to the fact that concrete hardening is caused by chemical reaction between cement and water which continues for a long period of time and consequently concrete get stronger with age (Gambhir, 2004). This gravel has round and smooth surface texture as compare to the conventional aggregate (granite) which possess irregular shape and can easily bind together as the aggregate key into one another more easily than the one with smooth surface Abbas el al. (2024).

Relationships between the Properties of Concrete Made from Bida Natural Coarse Aggregates

Empirical relationship between flexural strength (f_{cr}) and compressive strength (f_{cu})

Equations 5, 6 and 7 show the empirical relationship between compressive strength and flexural strength developed in this study.

$f_{cr} = 0.0757 f_{cu}$ (5)

$f_{cr} = 0.0336 \sqrt{f_{cu}}$ (6)

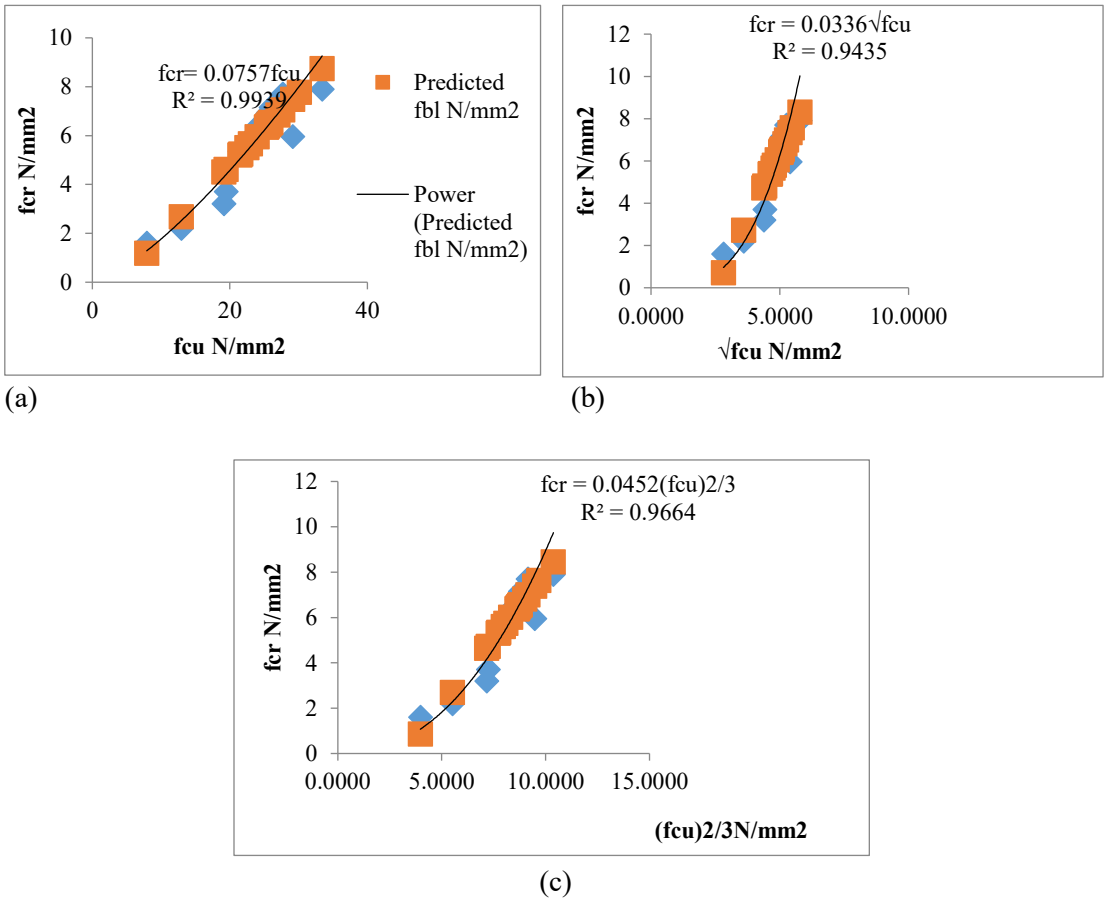
$f_{cr} = 0.0452 (f_{cu})^{2/3}$ (7)

Table 5 and fig 1 respectively portray the result and line fit plot for compressive strength and flexural strength.

A very high R^2 (coefficient of determination) was obtained for all the empirical relationship that was developed. This shows that all the equations interpret the experimental data satisfactorily. R^2 values of 0.9939, 0.9435 and 0.9664 were obtained for equations 5, 6 and 7 respectively, which indicate that 99, 94 and 97% of the variability in the flexural strength are explained by compressive strength for equation 5, 6 and 7 respectively.

The statistically significant equations were developed for flexural strength as a function of compressive strength. This is further proved by p - Value shown in Table 5 as 2.41E-09, 3.89E-09 and 2.71E-09 respectively which are far below 0.005 indicating high statistically significant of the model independent variable.

Table 5 also demonstrates the standard error for equation, 5, 6 and 7 as 0.68, 0.70 and 0.68 respectively. The errors are very minimal and insignificant. This further validate the ability of the models in predicting flexural strength of Bida gravel concrete from compressive strength. Moreover, a very slight deviation between experimental and predicated values for flexural strength were determined as -1.52 to 0.66, -1.5 to 0.9 and -1.43 to 0.75. It was observed from these three equations that equation 5, possessed the highest R^2 (0.9939), adjusted R^2 (0.86), lowest range of residual value (-0.45 to 0.19). Low p- value (2.41E-09) and lowest standard error (0.68). On that note the equation $0.0757f_{cu}$ best predict the flexural strength of Bida gravel concrete as a function of compressive strength. The chosen equation $0.0757f_{cu}$ is in line with equation developed as empirical relation between compressive strength and flexural strength by Abubakar *et al*. (2025).



(a) (b)

(c)

Figure 1**:** Line fit plots for flexural strength strength vs compressive strength

Table 5: Regression analysis for relationship between compressive strength and flexural strength

Equation form		**fcr= kfcu**		**fcr= k√fcu**		**fcr=k(fcu)$^{2/3}$**
	`R^2	0.9939	R^2	0.9434	R^2	0.9664
	AdjustedR^2	0.8606576	AdjustedR^2	0.85307059	AdjustedR^2	0.858792232
	Std. Error	0.6835445	Std. Error	0.70190713	Std. Error	0.688104807
	P-value	2.41E-09	P-value	3.89E-09	P-value	2.71E-09
Mix	Predicted	Residual	Predicted	Residual	Predicted	Residual
1	6.83	0.42	6.8	0.45	6.82	0.43
2	6.39	0.51	6.43	0.47	6.42	0.48
3	7.04	0.66	6.98	0.72	7	0.7
4	5.94	0.41	6.04	0.31	6.01	0.34
5	6.45	0.5	6.48	0.47	6.47	0.48
6	4.51	-1.31	4.7	-1.5	4.63	-1.43
7	5.65	0.25	5.78	0.12	5.74	0.16
8	6.34	0.46	6.39	0.41	6.37	0.43
9	5.19	0.16	5.36	-0.01	5.3	0.05
10	6.35	0.45	6.4	0.4	6.39	0.41
11	7.76	-0.21	7.55	-0.01	7.63	-0.08
12	4.61	-0.91	4.8	-1.1	4.73	-1.03
13	2.67	-0.47	2.72	-0.52	2.69	-0.49
14	6.51	0.49	6.53	0.47	6.53	0.47
15	7.47	-1.52	7.33	-1.38	7.38	-1.43
16	5.5	-0.1	5.65	-0.25	5.6	-0.2
17	1.18	0.42	0.7	0.9	0.85	0.75
18	5.23	0.02	5.4	-0.15	5.34	-0.09
19	8.74	-0.84	8.3	-0.4	8.45	-0.55
20	6.54	0.61	6.56	0.59	6.55	0.6

Empirical relationship between splitting tensile strength (f_{st}) and compressive strength(f_{cu})

Equation 8, 9 and 10 shows the empirical relationship between compressive strength and splitting tensile strength developed in this study.

fst = 0.0531fcu (8)

fst = 0.0354√fcu (9)

fst = 0.0404(fcu)$^{2/3}$ (10)

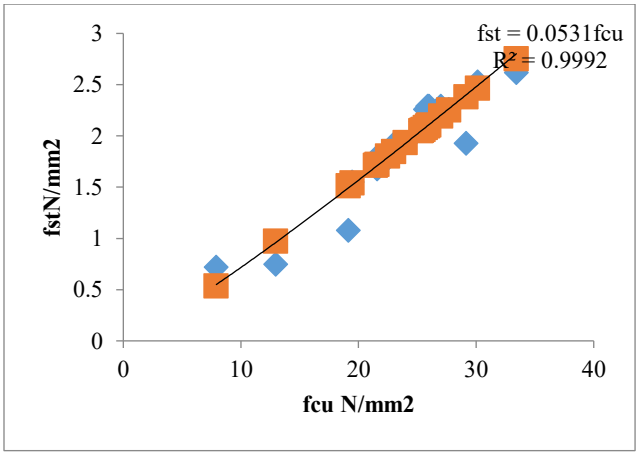
Table 6 and fig 2 respectively portray the result and line fit plot for compressive strength and splitting tensile strength.

A very high R^2 (coefficient of determination) was obtained for all the empirical relationship that was developed. This shows that all the equations interpret the experimental data satisfactorily. R^2 value of 0.9992, 0.9728 and 0.9858 were obtained for equation 8, 9 and 10 respectively, which indicate that 99, 97 and 99% of the variability in the splitting tensile

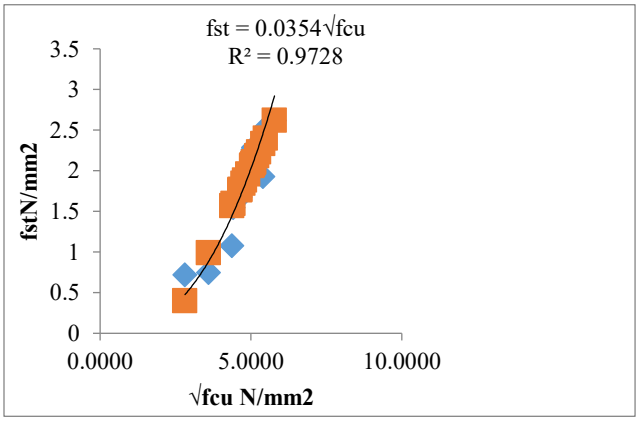
strength are explained by compressive strength for equation 8, 9 and 10 respectively. Thus, statistically significant equations where develop for splitting tensile strength as a function of compressive strength. This is further proved by p - Value shown in Table 6 as 9.57E-10, 2.79E-09 and 1.66E-09 respectively which are far below 0.005 indicating high statistically significant of the model independent variable

Additionally, Table 6 demonstrates the standard error for equation 8, 9 and 10 respectively as 0.18, 0.20 and 0.19 respectively. The errors are very minimal and insignificant. This further validate the ability of the models in predicting splitting tensile strength of Bida gravel concrete from compressive strength. Moreover, a very slight deviation between experimental and predicated values for flexural strength were determined as -0.45 to 0.19, -0.49 to 0.19 and -0.47 to 0.28. It was observed from these three equations that equation 8 possessed the highest R^2 (0.9992) adjusted R^2 (0.87), lowest range of residual value (-0.45 to 0.19). Low p- value (1.66E-09) and lowest standard error (0.19). On that note the equation 8 best predict the flexural strength of Bida gravel concrete as a function of compressive strength. The chosen equation 0.0531fcu is in line with equation developed as empirical relation between compressive strength and splitting tensile strength by Abubakar *et al*. (2025).

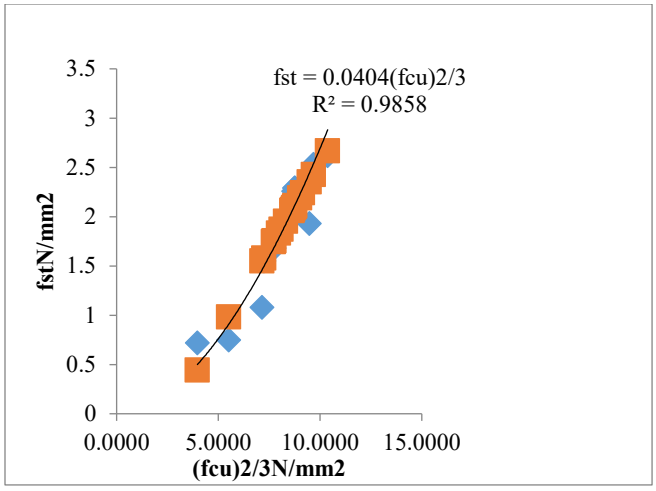
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(a)



(b)



(c)

Figure 2**:** Line fit plots for splitting tensile strength vs compressive strength

Table 6: Regression analysis for relationship between compressive strength and splinting tensile strength

Equation form	**f_{st}=Kfcu**		**f_{st}= k$\sqrt{f_{cu}}$**			**f_{st}= k(fcu)$^{2/3}$**
	R^2	0.999207	R^2	0.97286	R^2	0.9858
	Adjusted R^2	0.874133	Adjusted R^2	0.858383718	Adjusted R^2	0.866793647
	Std. Error	0.189088	Std. Error	0.200570876	Std. Error	0.194524248
	P-value	9.57E-10	P-value	2.79E-09	P-value	1.60E-09
Mix	Predicted	Residual	Predicted	Residual	Predicted	Residual
1	2.19	0.1	2.18	0.11	2.19	0.1
2	2.06	0.1	2.08	0.07	2.07	0.08
3	2.26	0	2.23	0.03	2.24	0.02
4	1.93	0	1.96	-0.03	1.95	-0.02
5	2.08	0.18	2.09	0.17	2.09	0.17
6	1.51	-0.43	1.57	-0.49	1.55	-0.47
7	1.85	0.07	1.89	0.03	1.87	0.05
8	2.05	0.09	2.06	0.08	2.06	0.08
9	1.71	0.07	1.76	0.02	1.75	0.03
10	2.05	0.09	2.07	0.07	2.06	0.08
11	2.47	0.06	2.4	0.13	2.43	0.1
12	1.54	0.01	1.6	-0.05	1.58	-0.03
13	0.97	-0.22	1	-0.24	0.98	-0.23
14	2.1	0.19	2.1	0.19	2.1	0.19
15	2.38	-0.45	2.34	-0.41	2.35	-0.42
16	1.8	-0.02	1.85	-0.07	1.83	-0.05
17	0.54	0.18	0.4	0.32	0.44	0.28
18	1.72	-0.04	1.77	-0.09	1.76	-0.08

19	2.75	-0.13	2.62	0	2.67	-0.05
20	2.11	0.18	2.11	0.18	2.11	0.18

Empirical relationship between elastic modulus (E) and compressive strength (f_{cu})

Equation 11, 12 and 13 shows the empirical relationship between compressive strength and elastic modulus developed in this study.

$E = 0.7886fcu$, (11)

$E = 0.6536\sqrt{fcu}$ (12)

$E = 0.6903(fcu)^{2/3}$ (13)

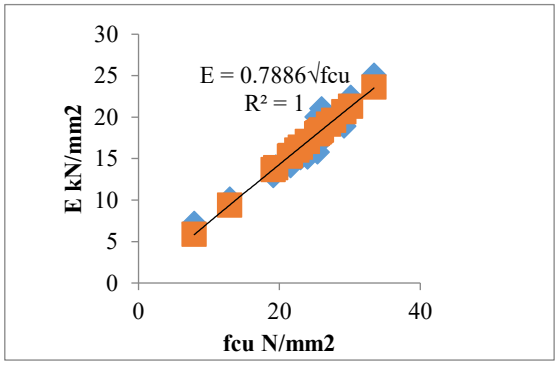
Table 7 and fig 3 respectively portray the result and line fit plot for compressive strength and flexural strength.

A very high R^2 (coefficient of determination) was obtained for all the empirical relationship that was developed. This shows that all the equations interpret the experimental data satisfactorily. R^2 value of 1, 0.9865 and 0.9943 were obtained for equation 11, 12 and 13 respectively, which indicate that 100, 99 and 99% of the variability in the splitting tensile strength are explained by compressive strength for equation 11, 12 and 13 respectively.

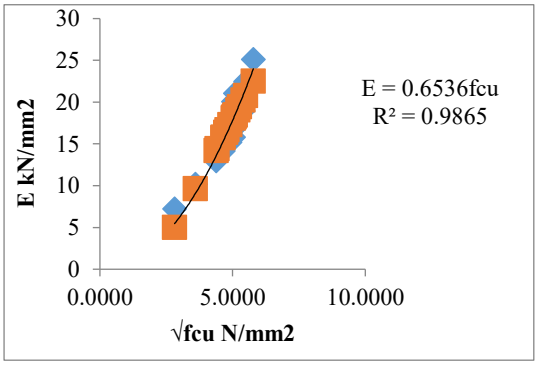
The statistically significant equations were developed for elastic modulus as a function of compressive strength. This is further proved by p - Value shown in Table 7 as 6.72E-10, 6.72E-10 and 9.95E-09 respectively which are far below 0.005 indicating high statistically significant of the modest independent variable.

Furthermore, Table 7 demonstrates the standard error for equation 11, 12 and 13 respectively as 1.72, 1.48 and 1.48 respectively. The errors are very minimal and insignificant. This further validate the ability of the models in predicting elastic modulus of Bida gravel concrete from compressive strength. Moreover, a very slight deviation between experimental and predicated values for elastic modulus were determined as -2.21 to 2.60, -2.21 to 2.60 and -2.30 to 2.61. It was observed from these three equations that equation 11 possessed the highest R^2 (1) adjusted R^2 (0.88), lowest range of residual value (-2.21 to 2.60). Low p- value (6.72E-10) and lowest standard error (1.42). On that note equation 11 best predict the elastic modulus of Bida gravel concrete as a function of compressive strength.

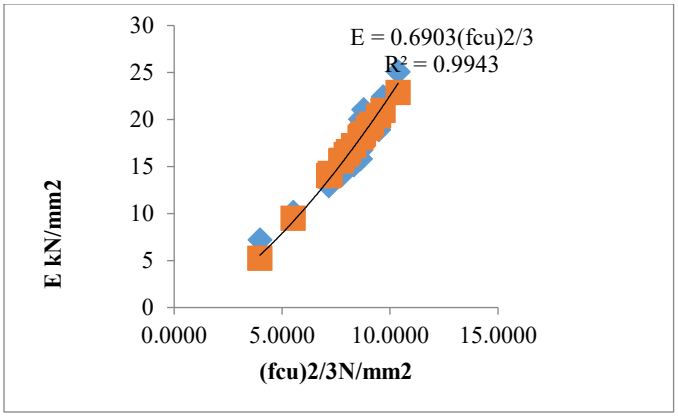
The chosen equation 0.7886fcu developed as empirical relation between compressive strength and elastic modulus is in line with the one developed by Abubakar *et al*. (2025).



(a)



(b)



(c)

Figure 3**:** Line fit plots for elastic modulus vs compressive strength

Table 7: Regression analysis for relationship between compressive strength and elastic modulus

Equation form		**E= kfcu**		**E= k$\sqrt{f_{cu}}$**		**E=k(fcu)$^{2/3}$**
	R^2	1	R^2	0.9865	R^2	0.9943
	Adjusted R^2	0.878954404	Adjusted R^2	0.878954404	Adjusted R^2	0.83706095
	Std. Error	1.718270571	Std. Error	1.480994529	Std. Error	1.48099452
	P-value	6.72E-10	P-value	6.72E-10	P-value	9.95E-09
Mix	Predicted	Residual	Predicted	Residual	Predicted	Residual
1	19.14	0.92	19.14	0.92	19.03	1.03
2	18.10	-1.33	18.10	-1.33	18.17	-1.40
3	19.63	0.43	19.63	0.43	19.43	0.63
4	17.06	-1.90	17.06	-1.90	17.28	-2.12
5	18.24	-0.92	18.24	-0.92	18.29	-0.97
6	13.69	-0.74	13.68	-0.74	14.19	-1.25
7	16.36	-1.28	16.36	-1.28	16.67	-1.59
8	17.98	2.06	17.98	2.06	18.07	1.97
9	15.29	-0.22	15.29	-0.22	15.71	-0.64
10	18.01	-2.21	18.01	-2.21	18.10	-2.30
11	21.32	1.11	21.32	1.11	20.76	1.67
12	13.91	0.10	13.91	0.10	14.42	-0.41
13	9.38	0.76	9.38	0.76	9.61	0.53
14	18.38	1.68	18.38	1.68	18.40	1.66
15	20.64	-1.73	20.64	-1.73	20.24	-1.33
16	16.01	-0.92	16.01	-0.92	16.36	-1.27
17	5.86	1.35	5.86	1.35	4.97	2.24
18	15.38	-1.25	15.38	-1.25	15.79	-1.66
19	23.61	1.48	23.61	1.48	22.48	2.61
20	18.44	2.60	18.45	2.60	18.46	2.59

CONCLUSION

Based on the result of this research the following conclusions were drawn;

i. The results of the physical properties of aggregate shows that it is suitable for concrete production having fulfil the requirements of relevant standard.
ii. The strength of concrete increase as the curing age increased.
iii. The relationship between the flexural strength and the compressive strength is expressed as $f_{bl} = 0.0757f_{cu}$.
iv. The relationship between the splitting tensile strength and the compressive strength is expressed as fst = 0.0531fcu.
v. The relationship between the elastic modulus and the compressive strength is expressed as 0.7886fcu
vi. The empirical equations developed are significant with high predictive capabilities and can be used in predicting the flexural and splitting tensile strengths of concrete.

Recommendations

It is recommended that the relationship between the flexural strength and the compressive strength, splitting tensile strength and the compressive strength, and elastic modulus and the compressive strength are expressed as $f_{bl} = 0.0757f_{cu,}$ 0.0531fcu and 0.7886fcu respectively.

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