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Reliability Based Optimum Design of Reinforced Concrete Flat Slab

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Abstract

In this study an analytical model was used to carry out reliability based optimum design of reinforced concrete flat slab using the sine cosine algorithm (SCA). An Interior flat slab panel was considered, in which the reliability and optimization formulation were derived based on Eurocode 2 (2004) using the prevalent at the time of the study. Design constraints were used to produce an optimal and reliable design, such as constraints associated with the reliability, upper and lower limits of slab depth and area of steel reinforcement. Slab depth and area of steel reinforcement are minimized to produce the optimal cost of the reinforced concrete flat slab while maintaining not less than acceptable level of reliability as specified by the Eurocode 0 (2002). Overall cost factor was developed to generalize and simplify the calculations of slab material cost of concrete and reinforcing steel of all sections and compared with the initial design. A reduction in design cost of the Reinforced concrete flat slab of about 9.03 % was achieved and The results clearly indicate that the optimum design parameters of the reinforced concrete flat slab can safely withstand the maximum imposed load of 5 kN/m^2 as recommended by EC 2 for high reliability class structures such as public. Numerical examples were presented to illustrate the model capability of calculating the reliability based optimum design cost of the reinforced concrete flat slab.

Key words: Optimum Design, Material Cost, Limit State Function, Sine Cosine Algorithm, Reinforced Concrete Flat Slab

Introduction

The common practice of design and construction is to support a slab by beams and support the beams by columns. This can be considered as beam slab construction, in which the beams reduce the available net clear ceiling height. As a result of that, in places like public halls, warehouses, offices and so on, sometimes beams are avoided and slabs are directly supported by concrete columns. These slabs which are directly supported by columns are called flat slab (Deshpande *et al.,* 2014). The slab can be of constant thickness in every part or, in the area of column it can be thickened as a drop panel. The column can also be of constant section or it may be flared to form a column head or capital (Figure1.1 (a & b)).

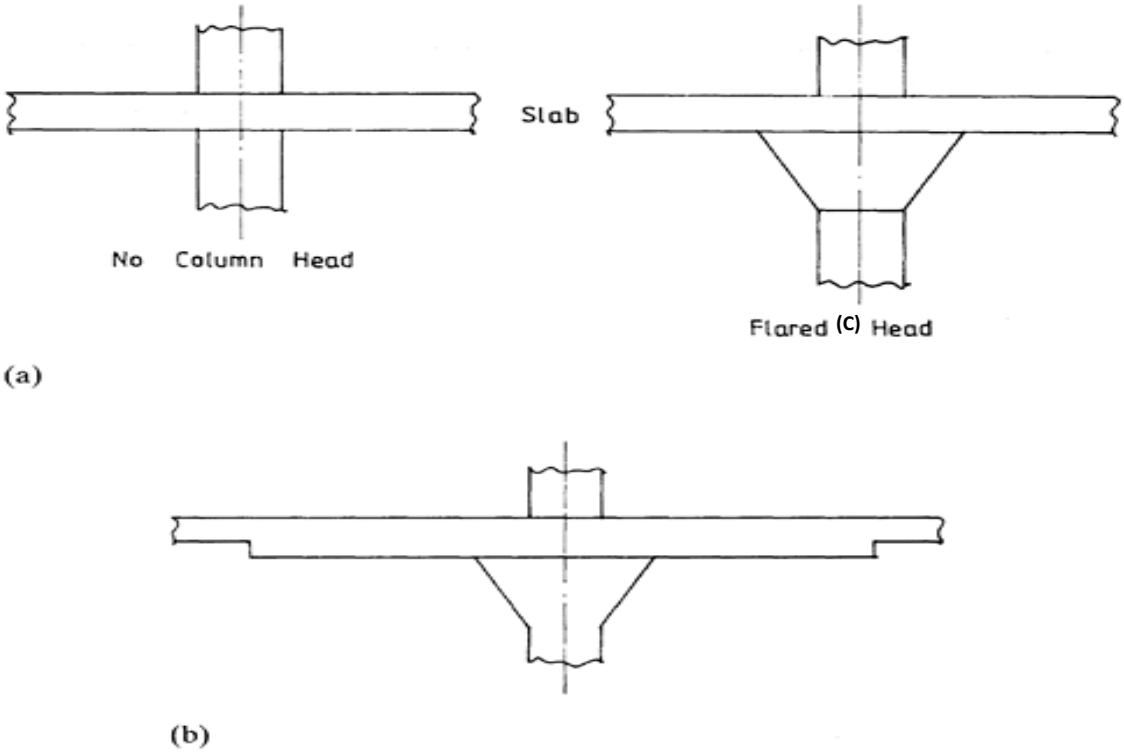


Figure 1: (a) Flat Slab without drop panel; (b) Flat slab with drop panel and flared column head, (c) Flat slab with column head. MacGinley and Choo (1990)

The fundamental reason to design is to have a unit or components which will not only be economical to construct but have some level of tolerance in terms of performance (Enevoldsen and Sorensen, 1994; Gouverneur *et al*., (2013).

Traditional design methods, such as those prescribed in normative codes, rely on deterministic values for loads, material strengths and resistance factors. This approach inherently assumes a level of safety through empirical factors of safety, but does not explicitly model the uncertainties inherent in structural systems. In contrast, reliability-based design treats key parameters (e.g., live loads, concrete strength,

reinforcement yield strength, geometric tolerances) as random variables and seeks to ensure that the probability of failure or the reliability index meets a target value. For example Elgohary et al. (2025) conducted a comparative reliability assessment of flat slab–edge column connections without shear reinforcement using Monte Carlo simulation. They found that the reliability indices of Eurocode 2, ACI 318, and Egyptian code formulations varied significantly, highlighting discrepancies in safety margins between international codes
Meanwhile, optimisation in reinforced concrete design is aim to minimise cost (material and labour), weight, subject to structural and serviceability constraints. In the case of flat slabs, optimisation is used to select slab thickness, reinforcement area, and concrete grade to minimise cost for given spans and loading. For example, Rady, Mahfouz & Taher (2022) developed an evolutionary algorithm to optimise floor systems (solid slabs, flat slabs with/without drop panels) and reported cost savings of up to 17% for the flat slab variant.
Structural engineers are not only interested in the design of safe systems but also the best systems (Arora, 2004). The design of the best system can be achieved using reliability based optimum design in which probabilistic approach are used in order to fully accommodate uncertainties in the optimization process. Probabilistic design permits a more rigorous quantification of the various uncertainties, and ultimately will facilitate a more efficient design process Enrique *et al*, (2003). Reliability analysis requires generation of the ultimate and serviceability limit state modes of failure which can be used to develop limit state functions and statistical parameters on the basic design variables.

Flat Slab Geometry

The reinforced concrete flat slab specifically analyzed in this study is directly supported by concrete columns at 6.5m centers in each direction and the slab supports a variable load of 5KN/m^2. The characteristic material strengths are f_{ck} = 25 N/mm^2 for concrete, and f_{yk} = 500 N/mm^2 for the reinforcement. The overall depth of the slab is 250mm and drop panels 2.5m square by 100 mm deep. The column heads are to be made 1.2 m diameter. The slab geometry was adopted from Mosley *et al*, (2006).

Structural Analysis

The slab was designed as an interior panel to resist punching shear, bending moment and deflection.The purpose of structural analysis is the establishment of the distribution of internal forces and moments, or stresses, and displacements, over the whole or part of a structure.

First Order Reliability Method (FORM)

In the first order reliability method (FORM), it is assumed that resistance (R) and load (S) are independent and both normally distributed (or have been transformed to normally distributed variables), then, the performance function (Z), supposed to be a linear function of normally distributed variables. This implies that the mean (μ) and variance (σ^2) of Z can be written as;

$\mu(Z) = \mu(R) - \mu(S)$ (1)

$\sigma^2(Z) = \sigma^2(R) + \sigma^2(S)$ (2)

The probability of failure follows from

$P\{Z<0\} = \int_0^\infty fz(\xi)d\xi = \Phi(-\beta)$ (3)

Where,

$\beta = \frac{\mu(Z)}{\sigma(Z)}$ (4)

Where β = safety or reliability index

Sine Cosine Algorithm (SCA)

This is a population-based optimization technique that starts optimization process with a set of random solutions. This random set is evaluated repeatedly by an objective function and improved by a set of rules which is the kernel of the optimization technique. Since population based optimization techniques look for the optima of optimization problems stochastically, there is no guarantee of finding a solution in a single run. However, with enough number of random solutions and optimization steps (iterations), the probability of finding the global optimum increases. Like other algorithms in the field of stochastic population- based optimisation, the process of this optimisation technique divided into two phases: exploration versus exploitation (Črepinšek et al, 2013). In the former phase, an optimization algorithm combines the random solutions in the set of solutions abruptly with a high rate of randomness to find the promising regions of the search space. In the exploitation phase, however, there are gradual changes in the random solutions, and random variations are considerably less than those in the exploration phase.

The following position updating equations are proposed for both phases:

$X_i^{t+1} = X_i^t + r_1 \text{ x } \sin(r_2) \text{ x } \left|r_3 P_i^t - X_i^t\right|$ (5)

$X_i^{t+1} = X_i^t + r_1 \text{ x } \cos(r_2) \text{ x } \left|r_3 P_i^t - X_i^t\right|$ (6)

Where X_i^t is the position of the current solution in a certain dimension at a certain number of iteration, $r_1/r_2/r_3$ are random numbers, P_i^t is position of the destination point in a certain dimension, and | | indicates the absolute value.

These two equations are combined to be used as follows:

$$X_i^{t+1} = \begin{cases} X_i^t + r_1 \text{ x } \sin(r_2) \text{ x } \left|r_3 P_i^t - X_i^t\right| & r_4 < 0.5 \\ X_i^t + r_1 \text{ x } \cos(r_2) \text{x } \left|r_3 P_i^t - X_i^t\right| & r_4 \geq 0.5 \end{cases} \quad (7)$$

Where r_4 is a random number in [0, 1]

The pseudo code of the SCA algorithm is presented in the following chart

Initialize a set of search agents (solutions)(X)
Do
    Evaluate each of the search agents by the objective function
    Update the best solution obtained so far (P=X*)
    Update r1, r2, r3, and r4
    Update the position of search agents using Eq. (3.11)
While(t< maximum number of iterations)
Return the best solution obtained so far as the global optimum

This chart above shows that the SCA algorithm starts the optimization process by a set of random solutions. The algorithm then saves the best solutions obtained so far, assigns it as the destination point, and updates other solutions with respect to it. Meanwhile, the ranges of sine and cosine functions are updated to emphasize exploitation of the search space as the iteration counter increases. The SCA algorithm terminates the optimization process when the iteration counter goes higher than the maximum number of iterations by default. However, any other termination condition can be considered such as maximum number of function evaluation or the accuracy of the obtained global optimum Seyedali (2016). The SCA based optimization routine has been programmed in MATLAB software.

The Limit State Function

The reliability-based optimum design formulation were developed based on the following limit-state function G(x), for the punching shear of reinforced concrete flat slabs based on Eurocode 2 (2004)

G(x) = G(R−S)

At the column head

$$G_{(X)} = (0.5\text{Ud}\,(0.6\left(1 - \frac{f_{ck}}{250}\right))\frac{f_{ck}}{1.5} - 1.15\left(F - \frac{\pi d_1^2}{4}\frac{F}{L^2}\right) \quad (8)$$

First Critical Section (2d from column face)

$$G(x) = 0.035\left(1 + \sqrt{\frac{200}{d}}\right)^{3/2} f_{ck}^{0.5}\,\text{U}d_2 - 1.15\left(F - \frac{\pi(d_1 + 4d_2)^2}{4}\frac{F}{L^2}\right) \quad (9)$$

At the drop panel

$$G(x) = 0.035\left(1 + \sqrt{\frac{200}{d_3}}\right)^{3/2} f_{ck}^{0.5}\,\text{U}d_3 - 1.15\left(F - A_P\frac{F}{L^2}\right) \quad (10)$$

Where d_1 = Column head diameter, d_2 = Effective depth of the slab at the support or within the drop panel, α = d_4 are the dimension of the square drop, d_3 = Effective depth of slab within the middle strip, R= Load, S= Resistance U = πd_1, F = design load = (1.35G_K + 1.5Q_K), L = Slab span, V_{ED} =1.15$\left(F - A_P \frac{F}{L^2}\right)$, A_P = $(d_4 + 3d_3^2)$ – (4− π)$2d_3^2$ and $U = 2d_4 + 2\ \alpha + 2\pi(2d_3)$

Limit State Function for Moment Capacity

In the middle strip

$$G_x = 0.87 f_{yk}\left[d_3 - \frac{A_{sm+}\, x\, f_{yk}}{b f_{ck}}\right] - 0.063 Q_k L_e \left(1.35\frac{G_k}{Q_k} + 1.5\right) \quad (11)$$

In the column strip

$$G_x = 0.87 f_{yk}\left[d_3 - \frac{A_{sc+}\, x\, f_{yk}}{b f_{ck}}\right] - 0.063 Q_k L_e \left(1.35\frac{G_k}{Q_k} + 1.5\right) \quad (12)$$

Where

A_{sm+}: Interior positive reinforcement in the middle strip, A_{sc+}: Interior positive reinforcement in the column strip,L_e = Effective span, b = width of the middle strip

Limit State Function for Deflection

$G_x = \left(\frac{L}{d}\right)$Limiting $- \left(\frac{L}{d}\right)$Actual

In the middle strip

$$G_x = \mathrm{K}\left(11 + \frac{1.5\sqrt{f_{ck}}}{\frac{A_{sm+}}{bd_3}} 10^{-3}\sqrt{f_{ck}} + 3.2\sqrt{f_{ck}}\left(\frac{10^{-3}\sqrt{f_{ck}}}{\frac{A_{sm+}}{bd_3}} - 1\right)^{1.5}\right) - \left(\frac{L}{d}\right)\text{Actual} \quad (13)$$

In the column strip

$$G_x = \mathrm{K}\left(11 + \frac{1.5\sqrt{f_{ck}}}{\frac{A_{sc+}}{bd_3}} 10^{-3}\sqrt{f_{ck}} + 3.2\sqrt{f_{ck}}\left(\frac{10^{-3}\sqrt{f_{ck}}}{\frac{A_{sc+}}{bd_3}} - 1\right)^{1.5}\right) - \left(\frac{L}{d}\right)\text{Actual} \quad (14)$$

Where ρo = Reference reinforcement ratio = $10^{-3}\sqrt{f_{ck}}$, K = 1.2 for flat slabs

Where f_{ck} = Characteristics strength of concrete, U = πd_1 , d = Effective depth at support, d_1 = Column head diameter, F = design load = (1.35G_K+1.5Q_K) in KN, L = Slab length

Formulation of objective function

The objective function for the cost was derived from equation 6

$C_T = C_C \times C_V + C_S \times W_S$ (15)

C_T = Total cost function, C_C = Cost of concrete per unit volume (Naira/m^3)
Cs=Cost of steel per unit mass (Naira/k_g), C_V = Concrete volume (m^3)
W_S = Weight of steel (k_g)

Depth constraint

The minimum thickness of the slab shall not be less than 125mm (H > 125mm)

$$g_1 = 1 - \frac{(1+ k_k)\ X\ d}{125} \leq 0 \quad (16)$$

Where k_k = Ratio of concrete cover to effective depth of slab, d = Effective depth of slab

Load constraint

$$g_2 = 1 - \frac{1.25G_k}{Q_k} \leq 0 \quad (17)$$

Area of steel reinforcement constraints

The area of reinforcement of the reinforced concrete flat slab (A_s) should be greater than or equal to the minimum area.

$$\mathbf{A_s \geq A_{min}} \quad (18)$$

$$g_3 = 1 - \frac{A_{sm}}{0.0013bd_3} \leq 0$$

$$g_4 = 1 - \frac{A_{sc}}{0.0013ad} \leq 0$$

d = Effective depth of the slab at the column strip, d_3 = Effective depth of slab in the middle strip, α = width of the column strip, b = width of the middle strip,Asm, Interior positive reinforcement in the middle strip, Asc, Interior positive reinforcement in the column strip.

Also, the area of reinforcement of the reinforced concrete flat slab should be less than or equal to the maximum area.

$$\mathbf{A_s \leq A_{smax}} \quad (19)$$

$$g_7 = 1 - \frac{0.04bD}{A_{sm}} \leq 0$$

$$g_8 = 1 - \frac{0.04aD_1}{A_{sc}} \leq 0$$

Where

b = width of the middle strip, D = overall depth of slab in the middle strip & α = width of the column strip, D_1 = overall depth of slab at support

Reliability index constraint

A reliability design constraint for each failure mode defined that the reliability index, β must be greater than its target value, β_T (i.e., a minimum level of reliability as recommended by the stated code).

$$\beta \geq \beta_T \quad (20)$$

Results and discussion

Figure 4 shows the output of the reliability based optimization, given the set of data from the program for the reinforced concrete flat slab. The result shows the optimum cost as 398,246 (naira), effective depth, d3 as 101.6mm and dead load, GK as 170.5(KN) with other parameters of the reinforced concrete flat slab design for over 400 iterations with a zero violation of constraints. While the betas (β) shown, defined as Beta_ps1= Safety index at the column head, Beta_ps2 = Safety index at the first critical section, Beta_ps3 = Safety index at the drop panel, Beta_d1 = Safety index of deflection along the middle strip, Beta_d2 = Safety index of deflection along the column strip, Beta_mc1 = Safety index of moment capacity at the middle strip, Beta_mc2 = Safety index of moment capacity at the column strip.

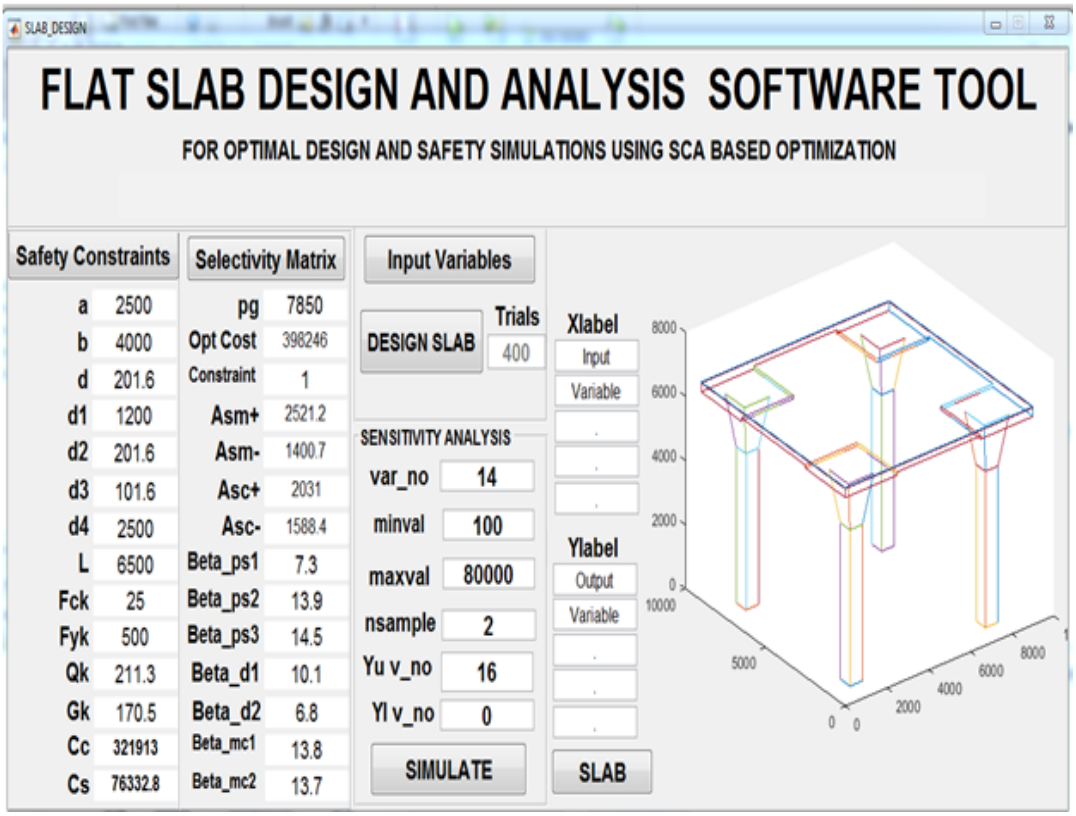


Figure 4: Output of reliability based optimization showing the optimum cost (naira), effective depth (mm), d3 and dead load (kN), Gk

Figure 5 shows the best convergence curve of the SCA on the slab design problem for the over 400 iterations after the algorithm run several times. The convergence curve is also an evidence and confirmation of the improvement of initial random population and the best solution obtained by SCA. The design improvement is quite significant in which the cost was reduced from 535,160 to 398,246 (Naira).

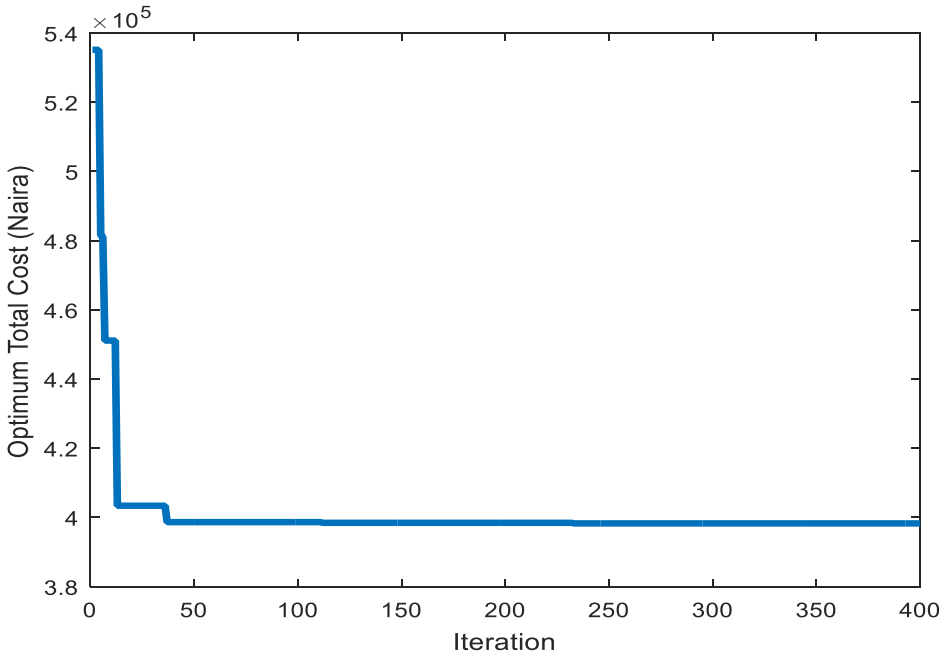


Figure 5: Convergence curve of the SCA on the optimum design of the reinforced concrete flat slab showing the initial and optimized cost

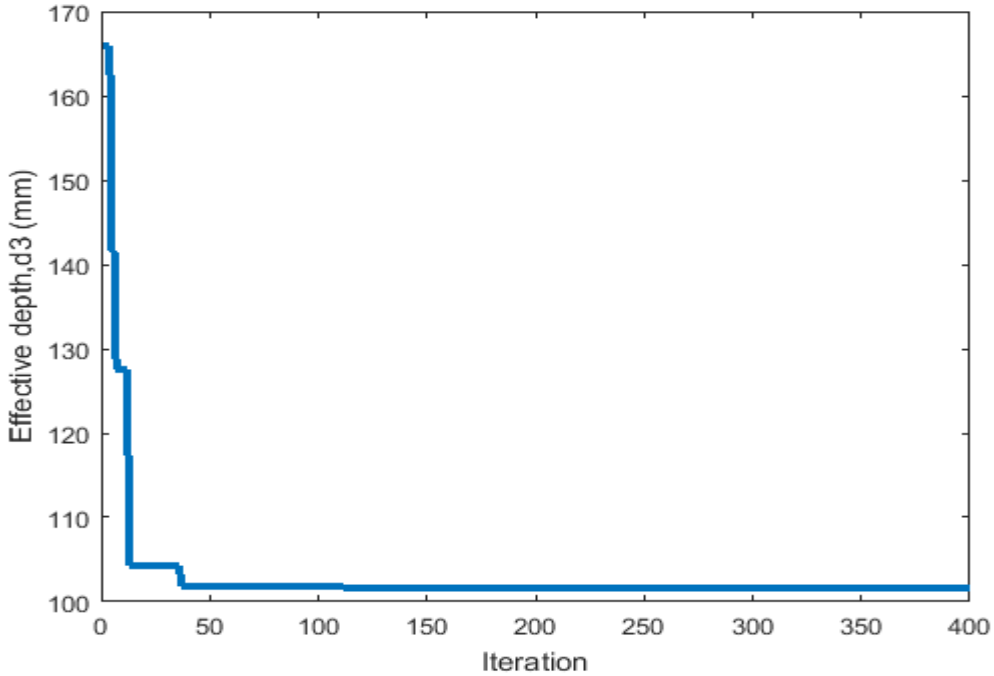


Figure 6: Convergence curve of the SCA showing the initial and optimized effective depth of the reinforced concrete flat slab

Figure 6 clearly shows that the Sine Cosine Algorithm improves the initial random effective depth of the slab to 101.6mm that led to the cost minimization at the end of the iteration.

Figure 7 shows how the dead load minimized to 170.5KN as a result of the minimization of the effective depth over same number of iterations.

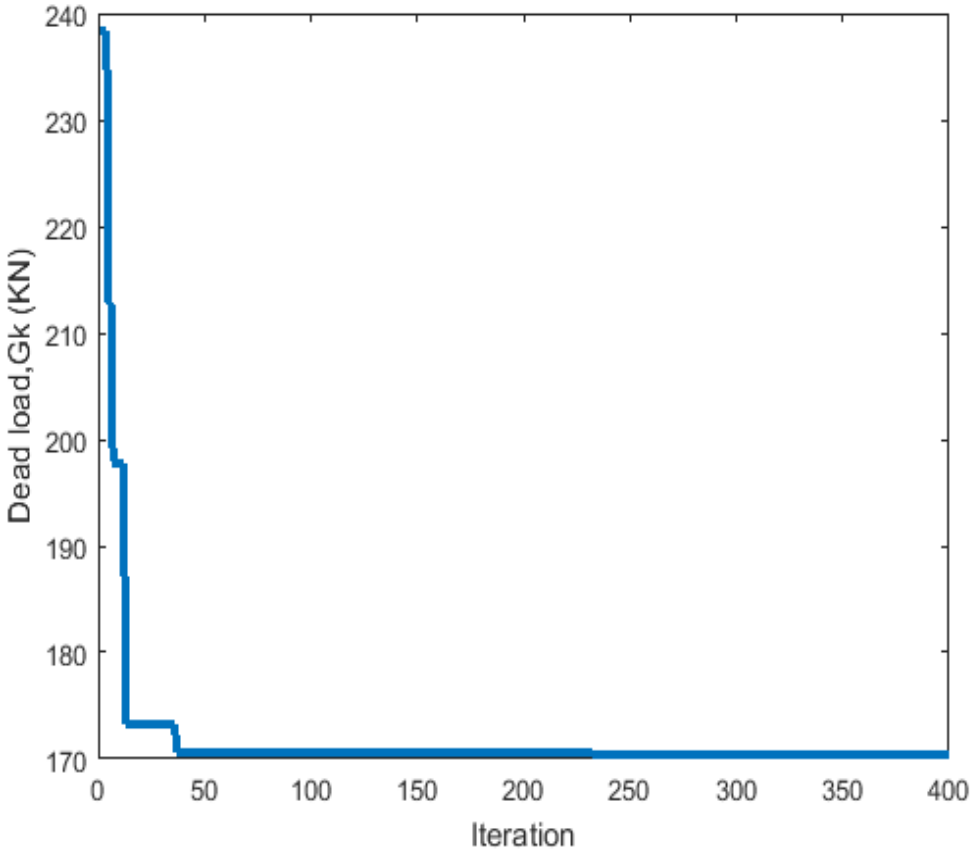


Figure 7: Convergence curve of the SCA showing the initial and optimized dead load of the reinforced concrete flat slab

Figure 8 & 9 show minimization and maximization of concrete cost and steel cost respectively while maintaining a higher reliability level to resist punching shear, deflection and bending moment over the same iteration. It was generally observed that as the slab thickness decreasing, the saving in the slab identified above, become exceeded by the extra reinforcement needed to improve or rectify performance deficiencies in the slab. However, the reduction in concrete volume (i.e concrete cost) outweighs the additional cost coming from increasing the steel section and cost.

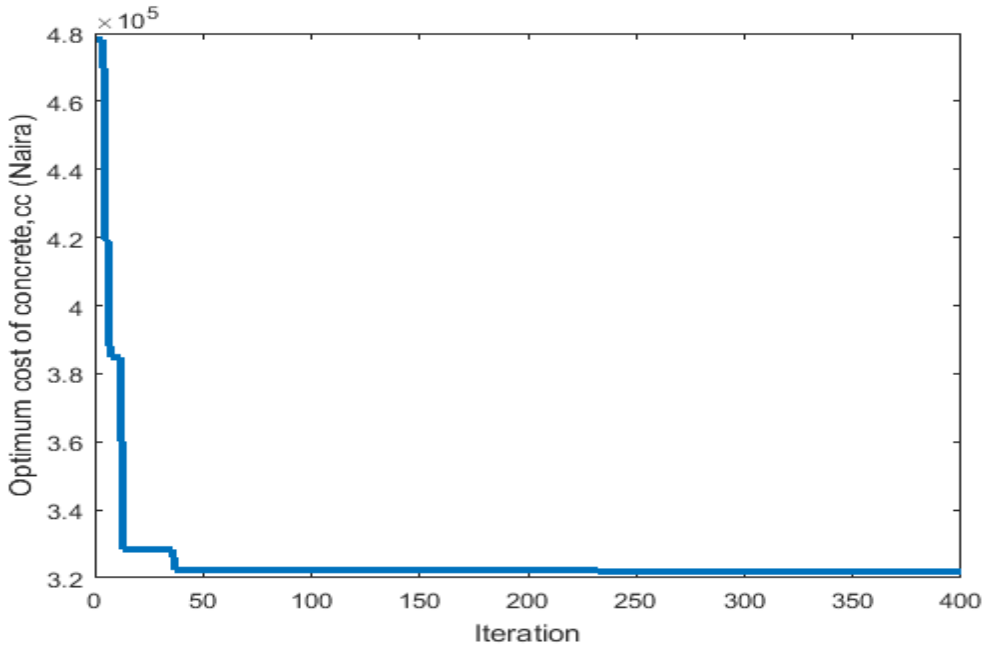


Figure 8: Convergence curve of the SCA showing the initial and optimized cost of concrete of the slab

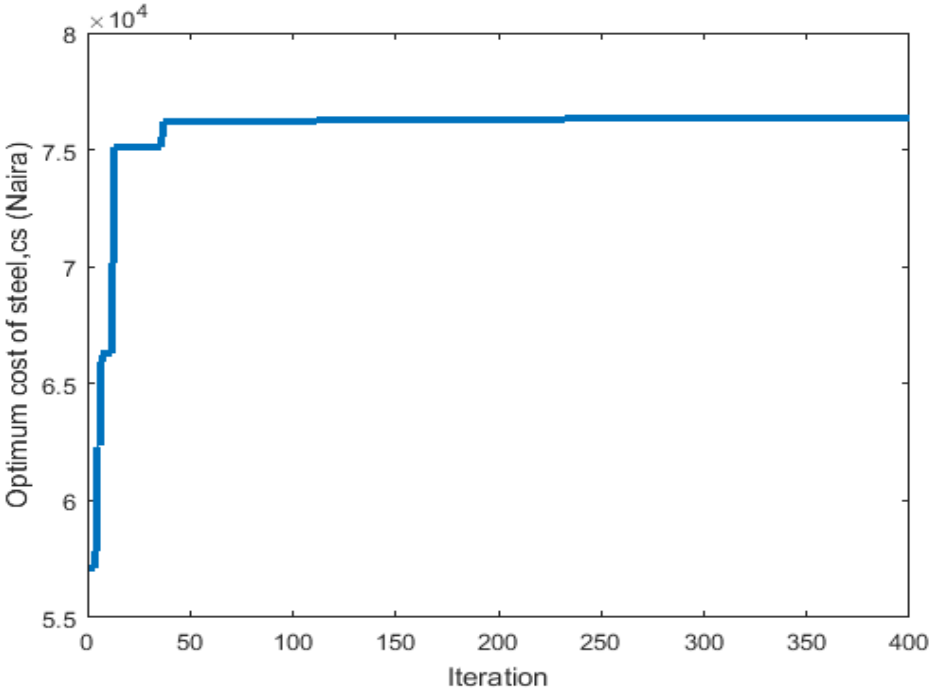


Figure 9: Convergence curve of the SCA showing the initial and optimized cost of steel in the slab

Conclusion

From analysis of the results of the reliability based optimum design of the reinforced concrete flat slab, the following conclusions were made;

1. For the five ultimate limit state modes of failures, the punching shear at the column head is discovered to be the least safe section and should be the leading section for punching shear design.
2. The result of the optimal cost observed to have achieved cost savings of 9.03% over the normal design of the reinforced concrete flat slab.
3. From the results obtained, it can be seen clearly that the reinforced concrete flat slab with optimized geometric parameters is capable of withstanding the maximum imposed load of 5 kN/m^2 as recommended by EC 0 for high reliability class structures such as public buildings

Recommendation

1. The optimum cost presented in this study was limited to only cost of concrete and steel reinforcement. There is need to consider cost of formwork and labour in the slab design.
2. Sensitivity analysis should be carried out to determine how the variations in input parameters affect the structural performance

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