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Predicting Population Growth in Taraba State, Nigeria: An Analysis Using the Runge-Kutta-Fehlberg Method

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Abstract

Accurate forecasting of population is crucial for effective policy development, resource management, and sustainable growth. This research utilizes the Runge-Kutta-Fehlberg (RKF) technique to project the population increase in Taraba State, Nigeria. Conventional models, including the Malthusian and logistic growth approaches, provide basic insights but often do not adequately reflect the complexities of actual demographic behavior. The logistic model, which takes into account the limits of carrying capacity, presents a more plausible framework for forecasting population patterns. The rate of prediction and the carrying capacity were estimated using data through MATLAB analytics, yielding values of $r \approx 0.0286$ and $K \approx 151{,}500{,}169$. By applying the RKF method, this study improves the precision of numerical solutions to the logistic model, thereby minimizing errors in long-term forecasts. By utilizing census data from Nigeria for the years 1991 and 2006, alongside projected figures from the National Population Commission, Nigeria, from 2007 to 2022, the RKF method yields dependable estimates of population trends through 2050. By 2050, it is anticipated that Taraba State's population will approach 8 million individuals. The findings illustrate the RKF method's capability to effectively capture demographic variations and enhance the accuracy of population forecasts. This study highlights the significance of sophisticated numerical approaches in demographic research and their potential to inform policy development and infrastructure planning in rapidly expanding areas like Taraba State.

Keywords: Population Growth, Runge-Kutta-Fehlberg Method, Logistic Growth Model, Demographic Forecasting, Taraba State, Nigeria

Introduction

The population dynamics of Taraba State, Nigeria, play a crucial role in guiding regional development, resource management, and urban planning. As state rich in ethnic diversity, fertile land and marked by steady population growth, accurate prediction of demographic trends is essential to support effective policy-making and sustainable development strategies. Traditional models, such as the Malthusian and logistic growth models, have provided foundational insights but often fall short of capturing the complexities inherent to real-world populations influenced by varied and changing factors.

The logistic growth model, introduced by Pierre François Verhulst (1838), offers a basic framework by incorporating concepts like intrinsic growth rates and carrying capacities to project population growth. However, in rapidly developing and dynamic regions like Taraba State, demographic patterns are shaped by a multitude of variables, including fluctuating birth and death rates, migration, economic factors, and resource availability, as highlighted by researchers such as Cohen (1995), Falola et al (2024) and George (2020). These complexities demand advanced modeling techniques to yield accurate and actionable forecasts, George (2020). Taraba State's population growth exhibits characteristics that align with logistic dynamics. Previous research suggests that incorporating stochastic elements can further improve forecasts. Studies such as Lan and Yanqiu (2020) and Mukesh and Ashish (2021) demonstrated that modifying logistic models with additional factors enhances real-world applicability. In the Nigerian context, refining the logistic equation with RKF improves predictive accuracy and policy relevance.

Overview of Taraba State

Established in 1991 from the former Gongola State, Taraba spans approximately 54,473 square kilometers, making it the third-largest state in Nigeria. With its capital in Jalingo and 16 local government areas (LGAs), the state boasts a diverse landscape from the fertile plains of riverine regions to the high-altitude Mambilla Plateau. This ecological diversity supports a range of agricultural activities, from staple crops like rice, maize, cassava and sorghum to cash crops such as sesame, egusi melon and groundnuts. The economy is primarily agrarian, with significant contributions from trade, tourism, and artisanal industries. Taraba's rich agricultural potential, combined with its vibrant local industries, makes population forecasting crucial for optimizing economic development, resource distribution, and social services across the state.

Taraba State shares borders with six Nigerian states and Cameroon, creating significant cross-border migration and trade opportunities. The presence of major rivers, including the Benue and Taraba Rivers, further supports agriculture, irrigation, and transportation, making population predictions vital for sustainable resource use and infrastructural planning.

Population Growth and Demographic Trends

Over recent decades, Taraba State has seen steady population growth, driven by both natural increase and migration. Challenges such as limited healthcare access, economic disparities, and educational gaps have

influenced fertility and mortality rates. Rural-to-urban migration has been notable, as people move to urban areas like Jalingo in search of better economic prospects. Understanding these demographic shifts is essential for managing urban expansion, resource allocation, and economic planning.

The Importance of Population Forecasting

The Nigerian National Population Commission is in charge of conducting a national census every ten years in accordance with a United Nations proposal and in accordance with international standard procedures of updating the nation's population data for efficient, well-informed policy making and planning. Nigeria's population census has remained a national concern over the years, linked to political manipulation and influence. Obstacles include inadequate finance and a lack of political will due to the president's declarations. According to Akanni (2020), these difficulties stem from past riots and rivalries for regional supremacy that dominated census administration. However, accurate forecasting is still the only way to estimate the population of Nigeria. Accurate population forecasting is key to ensuring that infrastructure, resources, and services align with the needs of Taraba's growing population. By assisting policymakers in anticipating and planning for future demands, forecasting models can promote sustainable development throughout the state when metropolitan areas undergo rapid growth. Since Taraba State was created in 1991, the only census data that is currently accessible is from the official Nigerian censuses in 1991 and 2006, which had 1,512,163 and 2,294,800 inhabitants, respectively. Because the Taraba State Population Commission only provided population statistics from 1991 to 2019, Samson and Shikaa (2023) pointed out that additional population forecasting is required. Thus, the population from 2007 up to 2050 is projected.

The Malthusian (1798) Growth Model and the Introduction of the Logistic Growth Model by Verhulst (1838)

The Malthusian growth model, formulated by Thomas Malthus in 1798, describes population dynamics as defined by the differential equation:

$$\frac{dP}{dt} = rP \quad \ldots \quad (1)$$

where P represents the population size and r the intrinsic growth rate. Solving this yields the exponential growth function:

$$P(t) = P_0 e^{rt} \quad \ldots \quad (2)$$

where P_0 is the initial population size. This model posits continuous, unchecked exponential growth under conditions of unlimited resources, a notion that underscores the inherent limitation of Malthus's approach: it does not consider environmental constraints. This limitation inspired further development of growth models that incorporate resource limitations.

In 1838, Pierre François Verhulst introduced the logistic growth model to address these limitations by including environmental carrying capacity, which caps the population size as resources become constrained. The logistic growth model is formulated as:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \quad \ldots \quad (3)$$

where K represents the carrying capacity of the environment. Solving this equation produces the population function:

$$P(t) = \frac{KP_0e^{rt}}{\left(K + P_0(e^{rt} - 1)\right)} \quad \ldots\ldots \quad (4)$$

where P_0 is again the initial population size. The logistic model, unlike the Malthusian model, illustrates population growth that decelerates as it approaches K, forming an S-shaped curve. This characteristic more accurately reflects real-world population dynamics, where growth is limited by finite resources.

Model Assumptions, Derivations, and Applications

Detailed derivations, assumptions, and applications for both models are well-documented in works by Murray (2020), May (2001), Turchin (2003), Shamsul (2024), Pierre (2022), and Gomez and Michael (2022). These resources extensively cover the mathematical underpinnings, real-world applications, and limitations of Malthusian and logistic models.

Recent studies have employed advanced numerical techniques to enhance the predictive power of these models. Mohammad et al. (2023), for instance, used a 6th Order Runge-Kutta (RK) method to solve equations (3) and (4) and forecast Bangladesh's future population. Similarly, Syarfiah et al. (2024) applied the Runge-Kutta Fehlberg (RKF) method to predict population growth in Sulbar, Madagascar, using logistic models. Their results underscored the accuracy and small error margin of the RKF method, suggesting its effectiveness for modeling population dynamics.

New Applications and Modifications of the Logistic Growth Model

Verhulst's logistic growth model is versatile, extending to various systems beyond single-species populations. Morales et al. (2023) utilized it to examine multiple species dynamics, while Windarto and Utami (2018) modified the logistic model to simulate growth patterns in poultry populations, achieving greater accuracy than traditional logistic, Richards, and Gompertz growth models.

Chusnul et al. (2024) evaluated the performance of the 4th Order Runge-Kutta (RK4) and Adams-Bashforth-Moulton (ABM4) methods on logistic equations, concluding that RK4 more accurately aligned with actual population data. Ekakitie and Ekereke (2019) applied a sensitivity model based on Malthusian theory to assess Nigeria's population growth over the long term, identifying crucial factors influencing growth such as carrying capacity and sensitivity coefficients. In order to increase its forecast accuracy, the logistic growth model has been modified to incorporate new components. Harvest elements were added by Lan and Yanqiu (2020) to enhance fish population prediction, producing a model with higher real-

world accuracy. The logistic model was modified by Mukesh and Ashish (2021) to accommodate the distinct demographic features of India, while Graham (2007) added a stochastic component to the logistic growth model to account for random fluctuations and provide a more complex understanding of population dynamics.

Numerical integration techniques are necessary to improve its precision, according to recent research conducted in Nigeria, such as those by Samson and Shikaa (2023). Furthermore, out of the logistic, Malthusian, geometric, and hyperbolic models, Nasir and Ibinayin (2020) showed that the logistic model yields the best accurate population estimates with the least amount of error. The Taraba State's population growth exhibits characteristics that align with logistic dynamics.

Methodology: The numerical Solution of Logistic Model for Population Prediction

In this study, the population of Taraba State is estimated numerically using the Runge-Kutta Fehlberg method, which is applied to a logistic population model. Taraba State's population statistics for 1991–2022 were sourced from Nigeria's National Population Commission via Nigeria Population Projection and Demographic Indicators-State and National (2020). The initial population, $P(t_0)$, is derived from the dataset. The dataset's graph representation is also used to determine the estimated growth rate (r) and carrying capacity (K). The logistic equation will be solve numerically using the Runge-Kutta-Fehlberg (RKF45) algorithm. To assess accuracy, the Runge-Kutta Fehlberg (RKF) method's numerical output will be contrasted with real census data. Graphical depictions of anticipated population trends are another way to visualize the numerical estimate.

Runge-Kutta-Fehlberg (RKF) Method

The Runge-Kutta-Fehlberg (RKF) method developed by Fehlberg (1996) offers a promising solution for addressing the variable dynamics of Taraba State's population growth. Renowned for its adaptive step-size mechanism, the RKF method is particularly effective for solving ordinary differential equations (ODEs) where demographic factors fluctuate. By allowing step-size adjustments based on the accuracy needed for each interval. The RKF method, an extension of classical Runge-Kutta methods, dynamically adjusts step sizes to reduce errors during periods of rapid demographic change, while maintaining efficiency during stable periods. This feature is essential for accurately modeling the demographic complexity of Taraba State, where migration patterns and resource availability vary significantly. The RKF's error control further enhances model reliability, particularly for long-term predictions where minor errors can lead to substantial discrepancies.

Below is the RKF schemes of order 4 and five respectively

Runge Kutta Fehlberg method of order 4 given as

$$P_{i+1} = P_i + \frac{25}{216} k_1 + \frac{1408}{2565} k_3 + \frac{2197}{4101} k_4 - \frac{1}{5} k_5 \quad \ldots \quad (5)$$

And Runge Kutta Fehlberg method of order 5 with better approximation.

$$\hat{P}_{i+1} = P_i + \frac{16}{135} k_1 + \frac{6656}{12,825} k_3 + \frac{28,561}{56,430} k_4 - \frac{9}{50} k_5 + \frac{2}{55} k_6 \quad \ldots \quad (6)$$

Where

$$k_1 = h f(t_i, P_i),$$
$$k_2 = h f\left(t_i + \frac{1}{4}h, P_i + \frac{1}{4}k_1\right),$$
$$k_3 = h f\left(t_i + \frac{3}{8}h, P_i + \frac{3}{3}2k_1 + \frac{9}{32}k_2\right)$$
$$k_4 = h f\left(t_i + \frac{12}{13}h, P_i + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3\right)$$
$$k_5 = h f\left(t_i + h, P_i + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4\right),$$
$$k_6 = h f\left(t_i + \frac{1}{2}h, P_i - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5\right)$$

Parameters Estimation

To accurately obtained the parameters for population projection, proper procedures needs to be followed. In this paper, the carrying capacity and the growth rate can be estimated using a Matlab codes on logistics model of fig. 1 below; while initial and the projected populations were obtained from Nigerian National Population Commission Projection 2022 for Taraba state.

Table 1. Taraba State Population Census 1991,2006 and projections from 2007 – 2022

Year	Population
1991	1,512,163
2006	2,294,800
2007	2,360,112
2008	2,428,624
2009	2,500,644
2010	2,576,092
2011	2,654,897
2012	2,736,961
2013	2,822,424
2014	2,907,564
2015	2,992,417
2016	3,076,898
2017	3,160,982
2018	3,245,094
2019	3,331,885
2020	3,421,510
2021	3,514,117
2022	3,609,843

Source: National Population Commission Projection, 2022

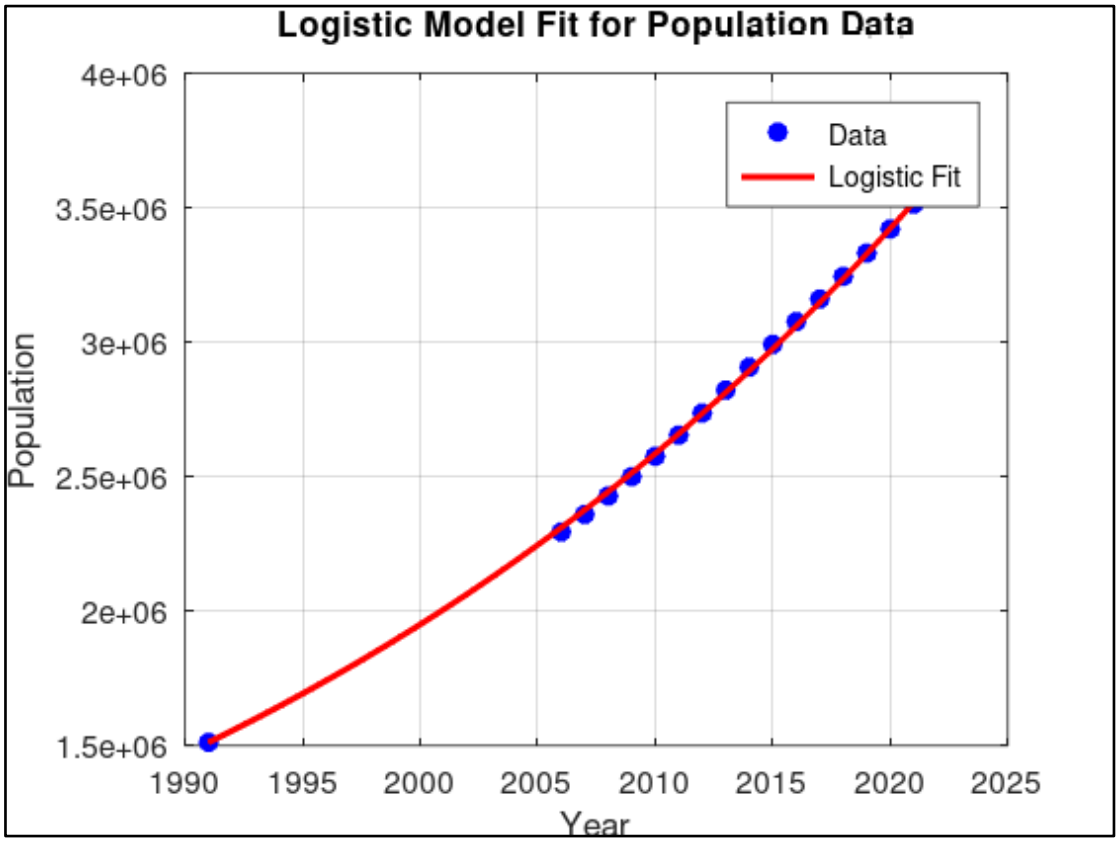


Fig. 1. Logistic fitted population raw data for Taraba State

From the data obtained from Nigerian National Population Commission Projection 2022; Taraba state from 2007 – 2022, the growth rate, r and the carrying capacity, K were estimated from Fig. 1, which is logistics fitted of Taraba State population.

Taraba State area $= 54{,}473$ km^2

Population at 1991 (initial population) $P_0 = 1{,}512{,}163$ people

Population rate of change $\approx 2.86\%$ or $r \approx 0.0286$

Estimated carrying capacity from the graph $K \approx 151{,}500{,}169$ people

Applying Runge Kutta Fehlberg methods to Taraba State Population

From equation (4), we have

$$\frac{dP}{dt} = rP(t)\left(1 - \frac{P(t)}{K}\right)$$

Where $\frac{dp}{dt}$ is the rate of change of population at time t, $P(t)$, is the population at time t and $P(t_0)$ is the initial population; K, is the carrying capacity and r, is the growth rate

Numerically, it can be represented as

$$f(t_i, P_i) = 0.0286 * P(t_i)\left(1 - \frac{P(t_i)}{151{,}500{,}169}\right) \quad \ldots \quad (7)$$

After substituting the values of r an K, where

$P(t_0) = 1{,}512{,}163$, is the initial population, which is 1991

$$k_1 = h f(t_0, P_0) = h\left(0.0286 * P(t_0)\left(1 - \frac{P(t_0)}{151{,}500{,}169}\right)\right) \quad \ldots \quad (8)$$

$$k_2 = h f\left(t_0 + \frac{1}{4}h, P_0 + \frac{1}{4}k_1\right) \quad \ldots \quad (9)$$

$$k_3 = h f\left(t_0 + \frac{3}{8}h, P_0 + \frac{3}{3}2k_1 + \frac{9}{32}k_2\right) \quad \ldots \quad (10)$$

$$k_4 = h f\left(t_0 + \frac{12}{13}h, P_0 + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3\right) \quad \ldots \quad (11)$$

$$k_5 = h f\left(t_0 + h, P_0 + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4\right) \quad \ldots \quad (12)$$

$$k_6 = h f\left(t_0 + \frac{1}{2}h, P_0 - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5\right) \ldots \quad (13)$$

$$P_{0+1} = P_1 = P_0 + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4101}k_4 - \frac{1}{5}k_5 \quad \ldots \quad (14)$$

And

$$\hat{P}_{0+1} = \hat{P}_1 = P_0 + \frac{16}{135}k_1 + \frac{6656}{12{,}825}k_3 + \frac{28{,}561}{56{,}430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6 \ldots \quad (15)$$

Where

$$t_1 = t_0 + h \quad \ldots \quad (16)$$

Using Matlab, the Runge-Kutta Fehlberg is used to simulates equations (7) to (16) to project Taraba State population from 1992 to 2050 as follows:

The Step by Step implementation of RKF simulation:

1. Define the differential equation as a function $f(t, P)$, the logistic model define in equation (7)
2. Initialize the initial population $P(t_0)$ for Taraba State which is 1991 population census figure, $P(t_0) = 1{,}512{,}163$ people
3. Set the initial step size, h
4. Calculate the various slopes k_1, k_2, k_3, k_4, k_5 and k_6, defined in equations (8) to (13)
5. Substitutes those value in RKF methods in (14) and (15) respectively and compute for the next periods of time
6. Implement RKF method in MATLAB programming
7. Run the simulation using the model to predict Taraba State population for 60 years.
8. Update population and adjust step size based on error estimate.
9. Visualize the population predictions by plotting graph and tables.

Table 2: Projected Taraba State Population from 2007 to 2050

Year	NPC, 2022	Projected Population using RKF45	Error (%)
1991	1,512,163	1,512,163	-
2006	2,294,800	2,309,900	0.66%
2007	2,360,112	2,375,900	0.67%
2008	2,428,624	2,443,700	0.62%
2009	2,500,644	2,513,400	0.51%
2010	2,576,092	2,585,100	0.35%
2011	2,654,897	2,658,800	0.15%
2012	2,736,961	2,734,500	-0.09%
2013	2,822,424	2,812,400	-0.36%
2014	2,907,564	2,892,400	-0.52%
2015	2,992,417	2,974,700	-0.59%
2016	3,076,898	3,059,300	-0.57%
2017	3,160,982	3,146,200	-0.47%
2018	3,245,094	3,235,500	-0.30%
2019	3,331,885	3,327,300	-0.14%
2020	3,421,510	3,421,700	0.01%
2021	3,514,117	3,518,600	0.13%
2022	3,609,843	3,618,300	0.23%
2025		3,934,000	
2030		4,520,800	
2035		5,191,900	
2040		5,958,700	
2045		6,833,400	
2050		7,829,600	

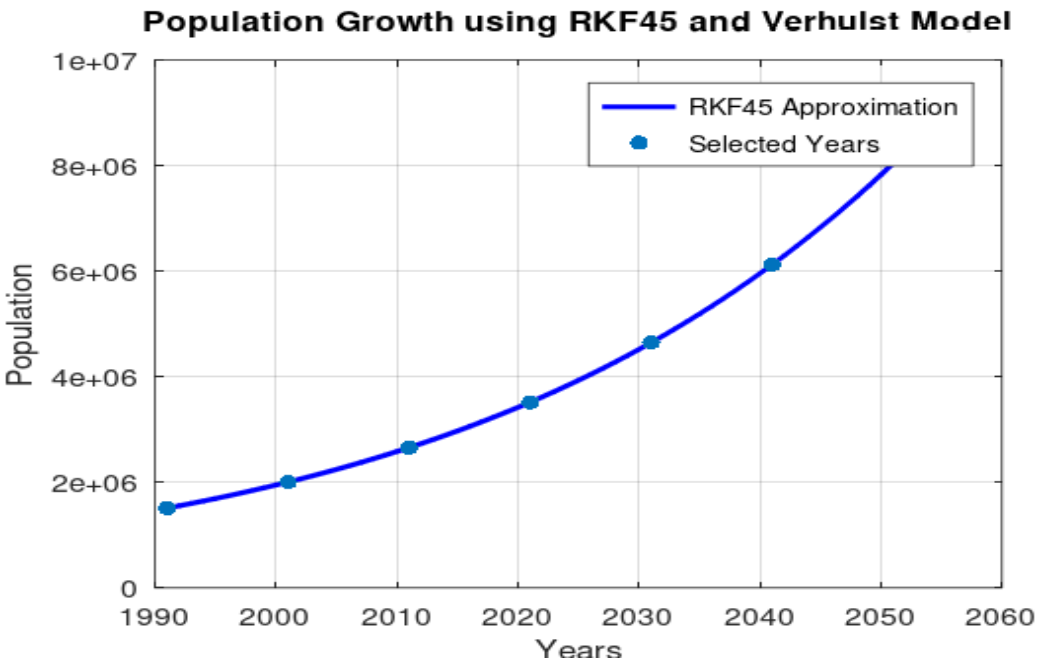


Fig. 2: Estimation Population projection using Runge-Kutta Fehlberg method RKF45 (Simulation in MATLAB)

Discussion

We can see from table 2 and figure 2 that the Runge-Kutta Fehlberg method is essential for determining Taraba State's population. The error is small and consistent, as can be shown, suggesting that the model is a reliable approximation. Additionally, given the rate of population expansion, the government must plan for the future. Policymakers and the government should come up with some significant development plans to accommodate Taraba State's expanding population in the future. According to the estimate, there would be a lot of societal issues if nothing is done for Taraba State's rapidly expanding population, which is expected to reach 8 million by 2050. Obstacles like unemployment, which reportedly causes disputes between communities, farmer-header disputes, kidnapping, etc.

Future Direction

Population growth is heavily impacted by socioeconomic factors like migration, education, healthcare availability, economic development, and governmental policies rather than being exclusively predicted by mathematical models. Migration from rural to urban areas has been a significant factor in Taraba State's demographic changes as people relocate in quest of improved job prospects. Changes in the economy have an effect on both birth and death rates; lower birth rates are frequently associated with greater economic stability, while lower death rates are associated with better healthcare. Furthermore, as higher educational attainment is linked to lower reproduction rates, access to education, especially for women is crucial for population management. Further influencing population patterns are government initiatives including social welfare programs, family planning laws, and infrastructure improvements. These socioeconomic factors can be added to forecasting models to increase their accuracy and make demographic projections more accurate. Socioeconomic indices should be incorporated into future research to improve logistic models and increase their prediction ability.

Conclusion

In conclusion, Planning and sustainable development depend heavily on population forecasts, especially in areas where demographic shifts are occurring quickly. To increase the numerical accuracy of population projections based on the logistic growth model, this study used the Runge-Kutta-Fehlberg approach. According to the results, the RKF approach yields more accurate estimates and successfully captures the demographic variances seen in Taraba State. The findings also underline the shortcomings of conventional models and the necessity of sophisticated computational methods in demographic studies. The study also recognizes the substantial influence of socioeconomic factors on population dynamics, including migration, economic stability, education, and governmental policy. This study helps to provide reliable forecasting tools that can assist policymakers in tackling future population concerns by fusing demographic modeling with numerical analysis.

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