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# Investigation of Some Heteroscedastic Models on Volatility Return under some Skewed-Distributed Error Innovations

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## *Abstract*

This study explores the performance of various heteroscedastic models in modeling and forecasting the volatility of financial returns, incorporating skewed-distributed error innovations to address the asymmetry and heavy tails observed in financial data. Volatility modeling is pivotal in financial econometrics, where understanding the variability of returns is essential for risk management, option pricing, and investment strategies. The investigation focuses on models from the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) family, including standard GARCH, Exponential GARCH (EGARCH), and Threshold GARCH (TGARCH), extended to accommodate skewed error distributions such as the Skewed Normal (SN), Skewed Student-t (SSt), and Skewed Generalized Error Distribution (SGED). Using empirical financial time series data, the models are compared based on their ability to capture volatility clustering, leverage effects, and asymmetric distributional properties of returns. Estimation of the

models is performed through maximum likelihood techniques, and their performance is evaluated using criteria such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and out-of-sample forecast accuracy. The results demonstrate that incorporating skewed-distributed error innovations significantly enhances the models' ability to capture the statistical properties of financial returns, leading to improved volatility forecasts. The findings highlight the importance of using skewed distributions in financial volatility modeling, providing more accurate tools for practitioners in risk assessment and financial decision-making. This study contributes to the ongoing development of more robust and flexible econometric models for financial markets.

**Keywords**: heteroscedastic models, volatility, skewed-distributed errors, financial returns, GARCH, EGARCH, TGARCH.

# Introduction

Volatility generally, is the rapid movement at which the value of a security increases or decreases over time. It is a rapid price change to know the best model for different errors.In financial markets, we are often concerned with the spread of asset returns. Statistically, volatility is often measured by the standard deviation. The importance and practical application of volatility modeling and forecasting in many finance applications means that the success or failure of volatility models will largely depend on the characteristics of empirical data that they try to capture and predict. Hence, the art of volatility modeling is to exploit the time series properties and stylized facts of financial market volatility. Financial time series have their unique characteristics; these several salient features about financial market returns and volatility now well documented include fat tails, volatility clustering, autocorrelation of squared returns, and no autocorrelation for asset returns except possibly at lag one. Thus, the specification of an appropriate volatility model for capturing these features exhibited by financial series is of significance to policy and economic managers. More so, a reliable volatility model of asset returns aids investors in their risk management decisions and portfolio adjustments.

An established feature of stock returns is that they exhibit volatility clustering i.e., periods of swing followed by periods of relative calm (Harvey and Chakravarty, 2008). In this regard, it

was discovered that, for vast classes of models, the average size of volatility is not constant but predictably changes with time. Interestingly, the generalized auto-regressive conditional heteroscedasticity (GARCH) model introduced by Bollerslev (1986) and Taylor (1986) is among the oldest and most widely used models for capturing volatility/risk, i.e., changing variances, in financial time series that exhibit time-varying volatility clustering. Also, studies on the volatility of stock returns which have been dominated by time series models of conditional heteroscedasticity have tended to find strong supports for asymmetric ARCH/GARCH-type effect. In Nigeria, however, the contributions of error distributions seem to have been ignored while modeling stock market volatility even though past and recent studies have shown that the application of appropriate error distribution in volatility models enhances the efficiency of a model (see e.g., Chang, 2010; Jiang, 2012). More so, the ARCH/GARCH–type models do not provide a theoretical explanation of volatility or what, if any, the exact contribution of information flows are in the volatility-generating process.Mantalos et al. (2019) study ARCH/GARCH effects under possible deviation from normality. Since skewness is the principal cause for deviations from normality in many practical applications, finance studies skewness in particular and proposes robust tests for normality for NoVaS and modified NoVaS transformed and original data. Such an approach is not applicable for EGARCH, but applicable for GARCH-GJR models. A novel test procedure was proposed for the skewness in autoregressive conditional volatility models. The power of the tests was investigated with various underlying models. Applications of financial data show the applicability and the capabilities of the proposed testing method.

**Statement of Problem**

The major assumption in the ordinary least square (OLS) in f is that it has a constant error variance over time which is known as homoscedasticity. However this assumption does not usually hold when dealing with financial series which is most of the real live data do not exhibit homoscedastic, therefore there is a need to find suitable heteroscedasticity. This problem led to the development of heteroscedastic models that can capture such data. Error innovation distribution plays a vital role in estimating the parameter of any volatility/ heteroscedastic models. The introduction of error distribution was first adopted by Engle (1982) where he

estimated the parameter of Autoregressive conditional heteroscedastic model ARCH using standard normal distribution as error distribution. On these error distributions, the normal distribution proposed by Engle (1982) gained more ground in the estimation of the volatility models. However, the traditional assumption of normality in modeling financial time series could weaken the robustness of parameter estimates. While modeling the volatility of returns, it is well known that financial return series exhibit many non-normal characteristics that cannot be captured by the standard ARCH or GARCH model with a normal error distribution. But which GARCH or other Heteroscedastic models and which error distribution to use is still open to question, especially where the model that best fits the in-sample data may not give the most effective out-of-sample volatility forecasting ability which we used as the criterion for the selection of the most effective model from among the alternatives.

Researchers have often modeled volatility with the normal error innovation over the years. Meanwhile, most financial series are non-normal, exhibit fat tails as well as highly leptokurtic, and some exhibit leverage effects. Thus, the use of normal error innovation for modeling data that exhibit these characteristics always yields inaccurate and poor forecast performance. Therefore, this study seeks to model appropriate theoretical alternative error innovation for (GARCH(p,q)) and EGARCH (p,q) models which are expected to improve and make better the deficiencies in the use of normal error innovation. The gap of the normal distribution which does not handle kurtosis of the error returns present in financial time series data has not been extensively filled by Agboola (2018). Just a few studies have been done in the area of error distributions. In this study, a new error distribution will be examined to account for the limitation of existing error distributions and used it to estimate some volatility models through simulation.

**Aim and Objectives**

This study aims to make a comparative study of some heteroscedastic models on volatility return with generalized and skewed-distributed error innovations.

The objectives of the study are to

***i.*** investigate the performance of some volatility models (ARCH,GARCH, EGARCH, and TGARCH) when the error terms are normally distributed;

***ii.*** investigate the forecast strength of the models at different error distributions and sample sizes

***iii.*** validly the models on a suitable financial data

**Literature Review**

One instance in which robust estimation should be considered is when there is a strong suspicion of heteroscedasticity. In the homoscedastic model, it is assumed that the variance of the error term is constant for all values of *x*. Heteroscedasticity allows the variance to be dependent on *x*, which is more accurate for many real scenarios. For example, the variance of expenditure is often larger for individuals with higher income than for individuals with lower incomes. The specification of appropriate volatility model for capturing fluctuations in stock returns is of significant policy relevance to investors and policy makers alike. First, reliable volatility model of asset returns aids investors in their risk management decisions and portfolio adjustments. The search for reliable volatility model is also important because unfettered fluctuation in asset return could influence investment decision that can manifest in the real sector with adverse consequences for economic growth and development. Second, undue swings in asset returns could impose challenges to monetary policy formulation and consequently undermine the price stability objective of the monetary authority. As De Gruwe (2008) argues, asset price inflation inevitably lead to crashes. In this regard, balance policy is needed since those policies that stabilize asset price can also stabilize inflation and output.

The general properties of financial time series that are called stylized characteristics become very important in applied economic analysis (Liu and Hung, 2010). Cont (2001) examined stylized. Emenike and Ani (2014) applied the GARCH model to the volatility of the banking sector indices in Nigeria.

Ogum et al. (2005) examined the volatility of Nigeria's stock market returns series. They fitted EGARCH model to the series, and the results showed the presence of asymmetric volatility in the Nigerian stock market. Olowe (2009) researched the relationship between stock market returns and volatility using an EGARCH-M model based on insurance and banking reforms, stock market collapse and the global financial crisis. The results revealed some evidence of relationship between volatility and stock returns, the impact of banking reforms and market crash was found to be negative and insurance reforms and financial crisis had no effect on stock returns. Emenike (2010) examined the leverage effects, volatility persistence and asymmetries of returns by fitting GARCH (1, 1) and GJR-GARCH (1, 1) models to the monthly NSE All-share-index. The findings of the study showed that returns process is characterized by fat-tail, leverage effects and volatility

persistence. Suleiman (2011) employed daily market capitalization index of the Nigerian stock market to assess the robustness of stock market returns volatility and its effect on capital market performance. The study utilized the ARCH/GARCH models to estimate the conditional variance of returns series. The findings from the study showed the presence of volatility in the conditional variance as well as long-term volatility persistence in the stock market. Kosapattarapim et al. (2022) evaluated the volatility forecasting performance of best fitting GARCH models in emerging Asian stock markets using the daily closing price data from Thailand (SET), Malaysia (KLCI) and Singapore (STI) stock exchanges by simulating six studies in GARCH(p, q) with six different error distributions. Findings from the simulations shows that the volatility forecast estimates of the best fitted model can be used for volatility forecasting confidently. Kalyanaraman (2014) examined the conditional volatility of Saudi stock market by employing AR (1)-GARCH (1, 1) model to the daily stock returns data. The findings of the study showed that returns process is characterized by time varying volatility, volatility clustering, persistence, and are predictable and also follows a non-normal distribution. AL-Najjar (2016) utilized ARCH, GARCH, and EGARCH models to investigate the behavior of Amman Stock Exchange (ASE) return volatility. The findings suggest that the symmetric ARCH /GARCH models are able to capture characteristics of ASE and also provide evidence of volatility clustering and leptokurtosis. Uyaebo et al. (2015) estimated asymmetric GARCH models with endogenous break dummy on two innovation assumptions using daily all share indexes of Nigeria, Kenya, United States, Germany, South Africa and China.

Researchers have often modeled volatility with the normal error innovation over the years. Meanwhile, most financial series are non-normal, exhibit fat tails as well as highly leptokurtic, and some exhibit leverage effect. Thus, the use of normal error innovation for modeling data that exhibit these characteristics always yields inaccurate and poor forecast performance. Therefore, this study seeks to model appropriate theoretical alternative error innovation for (GARCH (p,q)) and EGARCH (p,q) models which is expected to improve and make better the deficiencies in the use of normal error innovation. The gap of the normal distribution which does not handle kurtosis of the error returns present in financial time series data have not been extensively filled (by Agboola and Dikko, 2018). Just few studies have been done in the area of the error distributions. The study will be made used of the ARCH, GARCH, EGARCH and TGARCH models to

capture the volatility with heavy tails distribution of error terms that is present in financial time series.

**Methodology**

This study proposes the ARCH, GARCH, EGARCH and TGARCH models will be used to capture the volatility with heavy tails distribution of error terms that is present in financial time series. The assumption of homoscedasticity permits the use of the estimated regression equation to make forecast of the dependent variable. However, practically, all financial time series tend to exhibit varying variance; which vitiates the homoscedasticity assumption. Therefore, it becomes appropriate to consider frameworks that allow the variance to depend on its history.

In this study, four simulated studies on ARCH (p,q), GARCH (p,q),EARCH (p,q) and TGARCH models with four different error distributions (normal, student-t, skewed student-t, generalized error distribution) will be carried out at different sample sizes. In each case, the best fitting Heteroscedastic model based is determined on the MSE, MAE, and AIC criteria and then evaluates its out- of sample volatility forecasting performance against one another. The analysis will further be carried out on data from Nigeria stock exchange.

**Analytical Framework of ARCH/GARCH Family Models**

Every ARCH or GARCH family model requires two distinct specifications: the mean and variance equations. According to Engel, conditional heteroscedasticity in a stock return denoted by $y_t$ can be modeled using ARCH model expressing the mean equation in the form:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \cdots + \alpha_p y_{t-p} + \epsilon_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \epsilon_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

Equation 3.1 is a 'p' order autoregressive model. The mean equation is the same for all ARCH/GARCH family models.

$Where\ \alpha_0 > 0\ \epsilon_t = \sim N(0, \sigma^2)$. The residual ($\varepsilon_t$) is independent and identically distributed with a mean of zero and unit variance.

**The Autoregressive Conditional Heteroscedastic (ARCH) Models**

The conditional variance (volatility) is modeled with the use of ARCH (q) model by specifying the series $\sigma_t^2$thus: $\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \ldots\ldots\ldots (3.2)$

An ARCH (q) model can be estimated using ordinary least square (OLS). To test for the presence of ARCH effect (i.e., the presence of ARCH component), we need to check if the coefficient of the ARCH term (αi) is statistically significant or not. A finding of ARCH effect justifies the use of GARCH family models. We employ the Lagrange Multiplier test procedure proposed by Engle (1982) or the ARCH LM test. This procedure is as follows: First, we estimate the best fitting autoregressive model ARCH (q), and obtain the squares of the error $\epsilon^2$ and then regress them on a constant ($\alpha_0$) and q lagged values of the residual as follows

$$\hat{\epsilon}_t^2 = \alpha_0 + \sum_{i=1}^{q} \hat{\alpha}_i \epsilon_{t-i}^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

Where $\boldsymbol{q}$ is the lag order, i.e., length of ARCH lags. The hypothesis of ARCH effect is stated as follows:

$\boldsymbol{H_o}$: $\boldsymbol{a_i = 0}$ for all *i* = 1……..*q*. $\boldsymbol{H_1}$: $\boldsymbol{a_i \neq 0}$

The test statistic:

$$\frac{SSR_0 - SSR_1}{SSR_1(T - 2m - 1)}$$

Where

$$SSR_1 = \sum_{t=1}^{T} \epsilon_t^2 ,$$

$\epsilon_t^2 is\ the\ residual\ of\ least\ square\ of\ the\ linear\ regression\ and$

$$SSR_0 = \sum_{t=1}^{T} (a_t^2 - w), \qquad w = \frac{1}{T}\sum_{t=1}^{T} a_t^2$$

Is the sample mean of $a_t^2$

The decision rule for test of ARCH effect is as follows: If probability value is less than 5%, we reject $H_0$ (which say that there is no ARCH effect) and accept $H_1$. In other words, we conclude that there is an ARCH effect. As has been noted, the mean equation is the same for all ARCH/GARCH family models

**Generalized Autoregressive Conditional Heteroscedastic Model**

Bollerslev (1986) and Taylor (1986) Model If an autoregressive moving average model (ARMA model) is found for the error variance ($\sigma_t^2$), the model is a

generalized autoregressive conditional heteroskedasticity (GARCH) model. The general specification of GARCH (p, q) is as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \cdots + \alpha_q\epsilon_{t-q}^2 + \beta_1\sigma_{t-1}^2 + \cdots + \beta_p\sigma_{t-p}^2 = \alpha_0 + \sum_{i=1}^{q}\alpha_i\epsilon_{t-i}^2 + \sum_{i=1}^{p}\beta_i\sigma_{t-i}^2 \ldots\ldots\ldots (3.2)$$

Where $\alpha_i$ = coefficients of the ARCH component; and $\beta_i$ = coefficient of the GARCH component. The three parameter ($\alpha_0$, $\alpha_i$ and $\beta_i$) are restricted to be positive (assumed to be non-negative) and $\alpha_i + \beta_i < 1$, to achieve stationary. The test for the present of GARCH effect is established in two steps: First, we estimate the best fitting AR (q) model which is the AR (q) model with a lag order that gives the lowest Akaike and Swartz information and highest log-likelihood ratio.

Compute and plot the autocorrelations of $\hat{\epsilon}_t^2$ by

$$\frac{\sum_{t=i+1}^{T}(\hat{\epsilon}_t^2 - \hat{\sigma}_t^2)(\hat{\epsilon}_{t-1}^2 - \hat{\sigma}_{t-1}^2)}{\sum_{t=1}^{T}(\hat{\epsilon}_t^2 - \hat{\sigma}_t^2)^2}$$

**Exponential GARCH Model (EGARCH)**

(Nelson, 1991) The exponential generalized autoregressive conditional heteroskedasticity (EGARCH) model by Nelson (1991) is another version of the GARCH (p,q) model. The modification is that the lagged squared autoregressive component (ϵ2t-i) appearing in the standard GARCH model is replaced with a standard normal variable which may also come from a generalized error distribution (GED). Thus, the conditional variance of EGARCH (p,q) model is specified generally as:

$$\log(\sigma_t^2) = \beta_0 + \sum_{i=1}^{q}\left[\alpha_i\left|\frac{\epsilon_{t-i}}{\sigma_{t-i}}\right| + \gamma_i\left(\frac{\epsilon_{t-i}}{\sigma_{t-i}}\right)\right] + \sum_{j=1}^{p}\beta_j\log(\sigma_{t-j}^2)$$

**TGARCH Model**

The threshold-GARCH process allows analysis on the effects of good and bad news (negative and positive return shocks) on the volatility. TGARCH model is the standard GARCH models add up with the asymmetric threshold effect. The TGARCH model specification assumes that unexpected changes in the market returns will have different effect on the volatility of stock return, 2 V t. Good news will lead to higher return, and hence it is associated with higher variance through J. According to Ahmed & Suleiman (2011), a non-zero value of J indicates the asymmetric nature of the returns. On the other hand, when J is zero, we get back to

the standard symmetric GARCH model. When J is positive, there is a leverage effect.

**The Distribution of Error $\epsilon_t$**

Four different types of error distributions are considered in this study, these as follow;

*i*. Normal Distribution

$$f(z) = \frac{1}{\sqrt{2\pi}} exp\left(\frac{-z^2}{2}\right), \quad -\infty < z < \infty$$

*ii*. Student -t Distribution

$$f(z) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\Gamma\left(\frac{v}{2}\right)}\left(1 - \frac{z^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}, \quad -\infty < z < \infty$$

Where $\boldsymbol{v}$ denotes the number of degrees of freedom and Γ denotes the Gamma function.

*iii*. Skewed Student-t Distribution

$$f(z;\mu,\sigma,v,\lambda) = \begin{cases} bc\left(1 + \frac{1}{v-2}\left(\frac{b\left(\frac{z-\mu}{\sigma}\right)+\alpha}{1-\lambda}\right)^2\right)^{-\left(\frac{v+1}{2}\right)}, if\ z < -\frac{a}{b} \\ bc\left(1 + \frac{1}{v-2}\left(\frac{b\left(\frac{z-\mu}{\sigma}\right)+\alpha}{1+\lambda}\right)^2\right)^{-\left(\frac{v+1}{2}\right)}, if\ z \geq -\frac{a}{b} \end{cases}$$

Where *v* is a shape parameter with $2 < v < \infty$ and λ is a skewness parameter with $-1 < \lambda < 1$. The constants *a,b and c* are given below

$$a = 4\lambda c\left(\frac{v-2}{v-1}\right); \ b = 1 + 3\lambda^2 - a^2; \ c = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\pi(v-2)\Gamma\left(\frac{v}{2}\right)}$$

$\boldsymbol{\mu}$ And $\boldsymbol{\sigma^2}$ are the mean and variance of the skewed student-t distribution

*vi*. Generalized t Error Distribution

$$f(z;\mu,\sigma,v,\lambda) = \frac{\sigma^{-1} v e^{(-0.5(\frac{x-\mu}{\sigma})|\lambda|^v)}}{\lambda 2^{(1+1/v)}\Gamma\left(\frac{1}{v}\right)}, \quad 1 < z < \infty$$

$\boldsymbol{v > 0}$ is the degrees of freedom or tail -thickness parameter and

$$\lambda = \sqrt{2^{(-2/v)}\Gamma\left(\frac{1}{v}\right)\Gamma\left(\frac{3}{v}\right)}$$

If $\boldsymbol{v = 2}$, the GED yields the normal distribution. If ν < 1, the density function has thicker tails than the normal density function, whereas for $\boldsymbol{v > 2}$ it has thinner tails

**Evaluation, Comparison of Models and Selection Criteria**

At each scenario of specification, underline distributions and sample size, the models will be examined on the simulated data and compared using the finite sampling properties of models which are Akaike Information (AIC) criteria and Bayesian Information Criteria (BIC). The estimator with minimum criteria under different scenario of simulations will be taken as the best.

**Akaike Information Criterion (AIC)**

There are several information criteria available to determine the best model of autoregressive process. All of them are likelihood based; the well-known is Akaike information criterion (AIC) is a measure of the relative quality of a statistical model for a given set of data. That is, given a collection of models for the data, AIC estimates the quality of a model relative to each of the other models. Hence, AIC provides a means for model selection. Supposing we have a statistical model of some data, let k be the number of estimated parameters included in the model and $\boldsymbol{n}$, sample size, then, the AIC value of the model is

$AIC = nln(\hat{\sigma}^2) + 2k$ (See Alkaike, 1973 cited by Burnham and Anderson, 2001 and Tsay, 2010)

Where

$$\hat{\sigma}^2 = \frac{ResidualSumofSquares}{n}.$$

ln (likelihood) of the model given in the data, is readily available in statistical output, and reflects the overall fit of the model.

**Analysis and Results**

For the purposes of the analysis, the time series data on monthly exchange rates from the government, as well as the time series plots that display the observations on the y axis against equally spaced time intervals on the x axis used to evaluate

patterns and behaviour in data over time, are displayed in figures 1, the US Dollar and Pound Sterling exchange rates, respectively.

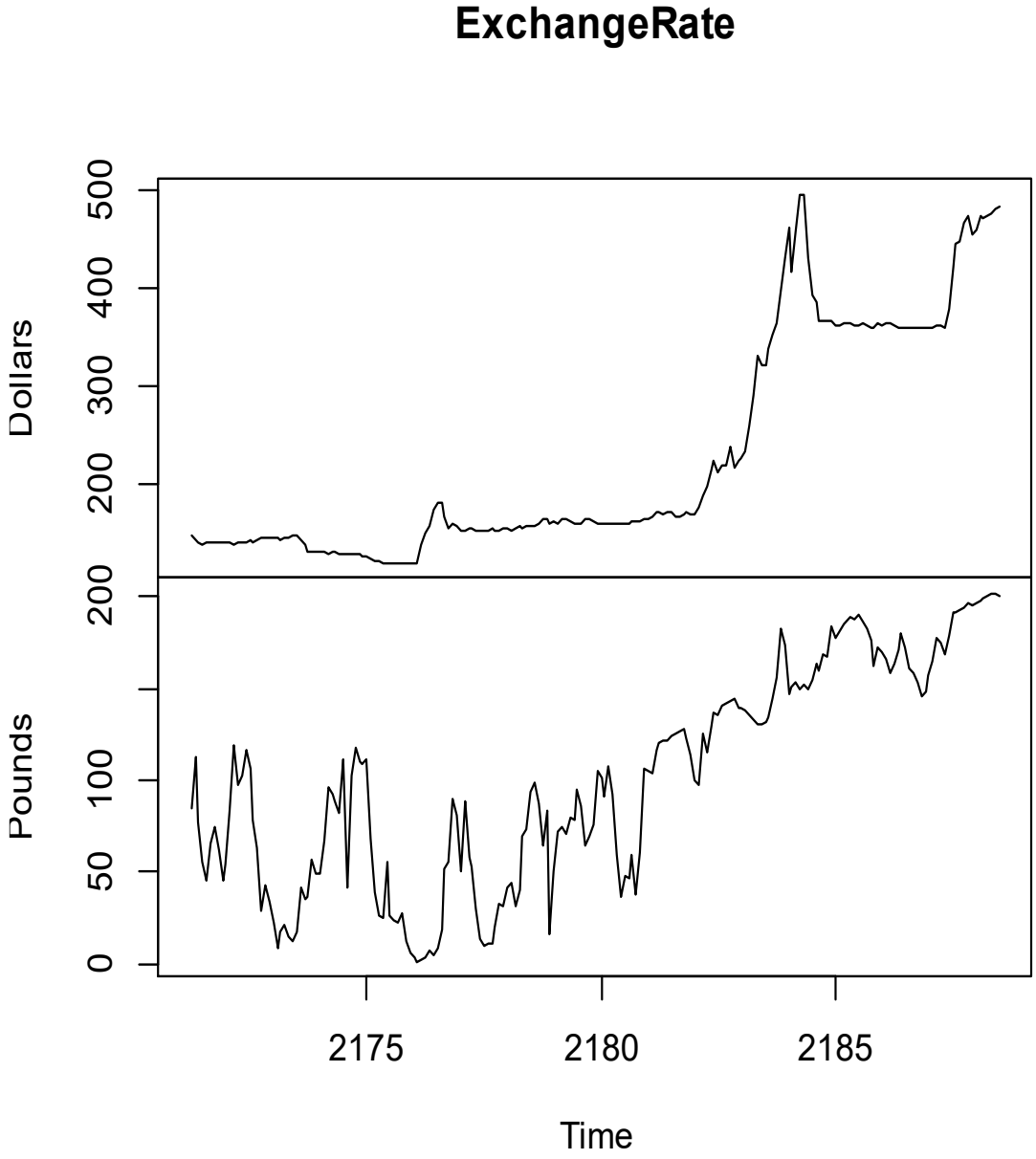



**Figure 4.8 Nigeria Exchange Rate of Dollars and Pounds Sterling**

Figure1. Clearly shows that the three exchange rates with respect to Naira were not constant, but rather varied from one year to the next as well as from one month to the next, with no systematically visible pattern, structural breaks, outliers, and no identifiable trend components in the time series data, or no consistently increasing or decreasing.

Table 1: Summary Statistics

| Exchange currency | Mean | Minimum | Maximum | Std. Dev | Skewness | Kurtosis | Jarque-Bera | P-Value |
|---|---|---|---|---|---|---|---|---|
| **Dollars** | 2.1215 | 0.0513 | -21.8951 | 4.7734 | 1.8953 | 8.9765 | 102.892 | 0.002 |
| **Pounds** | 2.7025 | 0.1041 | -69.661 | 3.7862 | 0.2397 | 8.1529 | 108.738 | 0.000 |

Table 1 summarizes the return series of the daily foreign exchange rate. According to the table, the anticipated return (mean) for the series is positive, implying that the Naira has seen more depreciation than appreciation against the US Dollar and Pound Sterling during the time under review. The standard deviation is likewise greater. The Dollar/Naira exchange rate has a high standard deviation, indicating that it is less stable. In terms of the distribution of the return series, the positive skewness (right tail) of the return series relative to the normal distribution (0 for the normal distribution) indicates a higher likelihood of Naira depreciation. Similarly, the kurtosis value indicates that the return series is leptokurtic (exhibit).

Table 2 Test for Unit Root (Stationarity) of the Exchange Rate**s**

| Test statistic | t-statistic | Level | Critical value | p-value | Decision |
|---|---|---|---|---|---|
| **Dollar** | -43.664 | 1% | -67.661 | 0.000 | The series is stationary |
| **Pounds** | -38.031 | 1% | -58.027 | 0.000 | The series is stationary |

The ADF statistic test in Table 2 evaluates the null hypothesis of a unit root to the alternative of no unit root, and the decision rule is to reject the null hypothesis when the value of the test statistic is less than the critical value. Because the ADF test statistic is greater than all critical values in absolute value, the hypothesis of non-stationary is rejected.

Table 3: Testing for ARCH Effects

| Test statistic | ARCH LM-test | Lag 2 | Lag 4 | Lag 6 | Decision |
|---|---|---|---|---|---|
| **Dollar** | Statistics | 14.891 | 78.3392 | 82.8876 | The series is |
| | P-Value | 2.021e-09 | 7.195e-16 | 2.092e-16 | heteroscedasticity |
| | P-Value | 1.001e-09 | 1.334e-16 | 1.005e-16 | |
| **Pounds** | Statistics | 15.889 | 74.985 | 90.125 | The series is |
| | P-Value | 2.775e-09 | 1.004e-16 | 1.005e-16 | heteroscedasticity |

The heteroscedasticity test findings for the return series are presented in Table 3. The null hypothesis that there is no ARCH effect is rejected for the return series.

Table 4: Ljung Box Q-Statistic

| | Ljung Box Q-Statistic | LB-Q | $LB-Q^2$ | Decision |
|---|---|---|---|---|
| **Dollar** | Statistics | 76.465 | 93.055 | There is higher order serial correlation |
| | P-Value | 2e-09 | 1.488e-16 | |
| | P-Value | 1.222e-09 | 2.02e-16 | |
| **Pounds** | Statistics | 85.005 | 93.777 | There is higher order serial correlation |
| | P-Value | 2.92e-09 | 1.07e-16 | |

Table 4's Ljung Box Q-Statistic for squared residuals demonstrates the significance of higher order serial correlation.

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Table 5: Heteroscedastic Models Selection on Volatility of Naira- US-Dollars Exchange Rates

| **Models** | **Equations** | **Parameters** | **Skewed Error (SGED** | **Generalized Distribution** | **Skewed Normal (SN)** | | **Skewed (SSt)** | **Student-t** |
|---|---|---|---|---|---|---|---|---|
| | | | **Coefficient** | **P-Value** | **Coefficient** | **P-Value** | **Coefficient** | **P-Value** |
| **GARCH** | Mean | MA (Θ) | 0.8864 | 0.0003 | -0.2621 | 0.0021 | -0.8671 | 0.0043 |
| **(1,1)** | Variance | Intercept(ϖ) | 0.1157 | 0.0006 | -2.4914 | 0.0024 | 0.0964 | 0.0046 |
| | | ARCH(α) | 0.3449 | 0.0009 | 2.7207 | 0.0027 | -0.3257 | 0.0049 |
| | | GARCH(β) | 0.5742 | 0.0012 | 2.9500 | 0.003 | -0.5550 | 0.0052 |
| | | Leverage(ϒ) | 0.8035 | 0.0015 | 3.1793 | 0.0034 | 0.7843 | 0.0055 |
| **Persistence** | | | 1.90223 | | 2.217.14 | | 3989.33 | |
| **Log-Likelihood** | | | 444.95 | | 2660.19 | | 4432.38 | |
| **AIC** | | | 887.997 | | 2103.23 | | 3875.42 | |
| **BIC** | | | 1331.04 | | 2546.28 | | 3318.47 | |
| **EGARCH** | Mean | MA (Θ) | -0.3171 | 0.0000 | -0.4881 | 0.0063 | 0.1082 | 0.0096 |
| **(1,1)** | Variance | Intercept(ϖ) | -0.6199 | 0.0411 | 0.6121 | 0.0067 | -0.2323 | 0.0005 |
| | | ARCH(α) | 0.7439 | 0.0455 | 0.7361 | 0.0072 | 0.3563 | 0.0046 |
| | | EGARCH(β) | -0.9920 | 0.0043 | 0.5975 | 0.0080 | 0.6043 | 0.0028 |
| | | Leverage(ϒ) | 0.1160 | 0.0587 | 1.9842 | 0.0019 | 0.1082 | 0.0096 |
| **Persistence** | | | 1.64651 | | 1.599.86 | | 3.97.17 | |
| **Log-Likelihood** | | | 401.199 | | 199.41 | | 1396.73 | |
| **AIC** | | | 800.752 | | 1398.96 | | 2796.28 | |
| **BIC** | | | 1200.3 | | 1798.52 | | 2195.83 | |
| **TGARCH** | Mean | MA (Θ) | -0.0879 | 0.0520 | -0.6118 | 0.00451 | 0.58709 | 0.0070 |
| **(1,1)** | Variance | Intercept(ϖ) | 0.1173 | 0.0073 | 0.3166 | 0.0040 | 0.6715 | 0.0073 |
| | | ARCH(α) | 0.5083 | 0.0027 | 0.6721 | 0.0035 | 0.7560 | 0.0077 |
| | | TGARCH1(β) | -0.1843 | 0.0001 | 0.2762 | 0.003 | 0.8405 | 0.003 |
| | | Leverage(ϒ) | 0.2513 | 0.0087 | 0.73858 | 0.0019 | 0.0095 | 0.0086 |
| **Persistence** | | | 1.84209 | | 1.731.34 | | 3.460.8 | |
| **Log-Likelihood** | | | 434.217 | | 2163.72 | | 2893.22 | |
| **AIC** | | | 866.593 | | 2596.09 | | 2325.6 | |
| **BIC** | | | 1298.97 | | 2028.47 | | 2757.97 | |

Table 5 provides estimates of the parameters of the GARCH, EGARCH, and TGARCH models on the exchange rate between the US dollar and the Naira at various error distributions. The estimations are determined to be significant for the return series at 5% significant levels for all fitted models. The necessity of modeling the dramatic and uncommon market events that have transpired during the sample period is demonstrated by the significance of these estimates. The fact that both a and ß are significant in the majority of the models suggests that news reports regarding volatility (or fluctuation) from earlier times can explain current volatility. Positive values for these coefficients indicate that volatility will likely rise as the market gets closer to the anticipated future rate, claim Longmore and Robinson (2004).

The coefficients of ß were relatively high for the models that were fitted. The GARCH(1,1), EGARCH(1,1), and TGARCH(1,1) models under the SGED, SN, and SSt error distributions produced high values of ß, which suggests that shocks to the conditional variance are persistent, whereas the low values of a like fitted show that volatility does not react swiftly to market changes. Additionally, the non-zero value of a for each of the models under consideration demonstrates a general feature of the models. The majority of the models' depictions of a seem to indicate that volatility clustering is present in the models. Additionally, when the absolute value of the standardized residuals is high (low), conditional volatility for these models tends to increase (down).

Since the variances are not stationary under the models, the estimated persistence coefficient ß for all the return time series is such that if the value is less than 1, as seen in the GARCH(1,1), EGARCH(1,1), and TGARCH(1,1), it indicates persistent volatility in the models and is necessary to have a mean reverting process. This suggests that the returns eventually move back toward the mean or average over time of the set of data considered; if the sum is greater than 1, of which none of the models used show this, it indicates that shocks to volatility are extremely high and will last forever. Based on the AIC and BIC model selection criteria, the best volatility model for the return series is observed to be the GARCH(1,1) since it has the lowest value for the selection criteria when compared to other models.

Table 6: Heteroscedastic Models Selection on Volatility of Naira and Pound Stally Exchange Rate

| Models | Equations | Parameters | Skewed Generalized Error Distribution (SGED | | Skewed Normal (SN) | | Skewed Student-t (SSt) | |
|---|---|---|---|---|---|---|---|---|
| | | | Coefficient | P-Value | Coefficient | P-Value | Coefficient | P-Value |
| **GARCH (1,1)** | Mean | MA (Θ) | 0.63275 | 0.0018 | -0.9165 | 0.0036 | 0.622 | 0.0058 |
| | Variance | Intercept(ϾϽ) | 0.66202 | 0.0021 | 0.1489 | 0.004 | 0.88714 | 0.0061 |
| | | ARCH(α) | 0.69129 | 0.0024 | 0.3814 | 0.0043 | -0.5228 | 0.0064 |
| | | GARCH(β) | 0.72056 | 0.0027 | 0.6138 | 0.0046 | 0.41742 | 0.0067 |
| | | Leverage(ϒ) | 0.74983 | 0.003 | 0.8462 | 0.0049 | 0.68256 | 0.007 |
| **Persistence** | | | 1.774.09 | | 2.217.15 | | 1.261.52 | |
| **Log-Likelihood** | | | 217.13 | | 2322.66 | | 3004.56 | |
| **AIC** | | | 1660.18 | | 2265.7 | | 3875.42 | |
| **BIC** | | | 1103.23 | | 2208.75 | | 2747.61 | |
| **EGARCH (1,1)** | Mean | MA (Θ) | -0.3171 | 0.0000 | -0.4881 | 0.0063 | 0.1082 | 0.0096 |
| | Variance | Intercept(ϾϽ) | -0.007 | 0.0403 | 0.9769 | 0.0025 | 0.5265 | 0.0107 |
| | | ARCH(α) | -0.0229 | 0.04146 | 0.37467 | 0.0011 | 0.65345 | 0.0133 |
| | | EGARCH(β) | 0.02431 | 0.04262 | 0.77244 | 0.0002 | 0.7804 | 0.0158 |
| | | Leverage(ϒ) | 0.07149 | 0.04378 | 0.17021 | 0.0015 | 0.90735 | 0.0184 |
| **Persistence** | | | 1.599.85 | | 1.599.87 | | 3.9718 | |
| **Log-Likelihood** | | | 1999.4 | | 2731.41 | | 2928.71 | |
| **AIC** | | | 2398.96 | | 3530.96 | | 3328.26 | |
| **BIC** | | | 2798.51 | | 2330.52 | | 3727.81 | |
| **TGARCH (1,1)** | Mean | MA (Θ) | 0.27166 | -0.0234 | 0.34248 | 0.0008 | 0.17845 | 0.0062 |
| | Variance | Intercept(ϾϽ) | 0.30934 | 0.0327 | 0.60852 | 0.0002 | 0.07983 | 0.0061 |
| | | ARCH(α) | 0.34702 | 0.0421 | 0.87455 | 0.0004 | 0.0188 | 0.0060 |
| | | TGARCH1(β) | 0.3847 | 0.0515 | 0.14059 | 0.001 | 0.1174 | 0.0059 |
| | | Leverage(ϒ) | 0.42238 | -0.0609 | 0.40662 | -0.0016 | 0.2164 | 0.0058 |
| **Persistence** | | | 2.731.3 | | 3.3258 | | 3.460.9 | |
| **Log-Likelihood** | | | 1063.7 | | 1077.2 | | 10023.7 | |
| **AIC** | | | 10096.1 | | 1008.6 | | 1006.1 | |
| **BIC** | | | 10008.5 | | 11209.9 | | 10388.4 | |

Table 6 presents estimates of the parameters of the GARCH, EGARCH, and TGARCH models on the exchange rate between the Naira and Pound Stally at various error distributions. The estimations are determined to be significant for the return series at 5% significant levels for all fitted models. The necessity of modeling the dramatic and uncommon market events that have transpired during the sample period is demonstrated by the significance of these estimates. The fact that both a and ß are significant in the majority of the models suggests that news reports regarding volatility (or fluctuation) from earlier times can explain current volatility. Positive values for these coefficients indicate that volatility will likely rise as the market gets closer to the anticipated future rate, claim Longmore and Robinson (2004).

The coefficients of ß were relatively high for the models that were fitted. The GARCH(1,1), EGARCH(1,1), and TGARCH(1,1) models under the SGED, SN,

and SSt error distributions produced high values of ß, which suggests that shocks to the conditional variance are persistent, whereas the low values of a like fitted show that volatility does not react swiftly to market changes. Additionally, the non-zero value of a for each of the models under consideration demonstrates a general feature of the models. The majority of the models' depictions of a seem to indicate that volatility clustering is present in the models. Additionally, when the absolute value of the standardized residuals is high (low), conditional volatility for these models tends to increase (down).

Since the variances are not stationary under the models, the estimated persistence coefficient ß for all the return time series is such that if the value is less than 1 (<1), as seen in the GARCH (1,1), EGARCH (1,1), and TGARCH(1,1), it indicates persistent volatility in the models and is necessary to have a mean reverting process. This suggests that the returns eventually move back toward the mean or average over time of the set of data considered; if the sum is greater than 1, of which none of the models used show this, it indicates that shocks to volatility are extremely high and will last forever. Based on the AIC and BIC model selection criteria, the best volatility model for the return series is observed to be the TARCH (1,1) since it has the lowest value for the selection criteria when compared to other models.

## Conclusion

The GARCH (1,1) produced better estimates of the parameters on the daily data of the Naira/Dollar and Pounds sterling exchange rates for the period of 2000 to 2024 to model the volatility of exchange rate returns, when compared to the corresponding estimates produced by other respective models, this conclusion made is based on the smallest value of the AIC and BIC produced by the analyses carried out in the study. Also, the TGARCH (1,1) produced better estimates of the parameter on Naira/ Pound exchange rate when compared to the corresponding estimates produced by other respective models It has been established in this work that GARCH(1,1) and TGARCH(1,1) as regards AIC and BIC respectively would readily capture all the stylized fact of financial data such as persistence, volatility clustering, and asymmetric effect particularly when the data is non-normal and leptokurtic in nature.

## References

Ade I and Dallah, H. (2020). *"Modelling and Forecasting the Volatility of the Daily Returns of Nigerian Insurance Stocks International Business Research 13(2): 106-116.*

Agboola, S. , Dikko, H. G. and. Asiribo O. E (2018). *On Exponentiated Skewed Student t Error Distribution on Some Heteroscedastic Models: Evidence of Nigeria Stock Exchange. J. Stat. Appl. Pro. 7, No. 1, 205- 216*

*Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics, 31 (3): 307–327*.

Akaike, H. (1973). *Maximum likelihood identification of Gaussian auto-regressive moving-average models." Biometrika, 60, 255–266.*

AL-Najjar, D. (2016). *Modelling and Estimation of Volatility Using ARCH/GARCH Modelsin Jordan's Stock Market. Asian Journal of Finance & Accounting, 8(1), 152-167.*

xArestis, P; Demetriades, P. O. and Luintel, K.B. (2001). *"Financial Development and Economic Growth: The Role of Stock Markets", Journal of Money, Credit and Banking, 33 (2) pp 16-41*

Bala, D. A., & Asemota, O. J (2013). *Exchange Rates Volatility in Nigeria: Application of GARCH Modes with Exogenous Break. CBN Journal of Applied Statistics, 4(1), 89-116.*

Bollerslev, T. and Taylor, I. (2021). *Price volatility, spread variability and the role of alternative market mechanisms, The Review of Future Markets, 10(1), 78-102.*

Burhnam, K. Pand Anderson, D. R. (2001). *Model selection and inference: a practical information theoretic approach. New York: Springer*

Chang, S. (2010). *Application of EGARCH Model to Estmate Financial Volatility of Daily Returns: The Empirical Case of China. School of Business, Economics and Law, University of Gothenburg*.

Cont, R. (2001). *Empirical properties of asset returns: stylized facts and statistical issues. Quantitative Finance, 1, 223-236*

Dallah, H. and Ade I. (2020). *Modelling and Forecasting the Volatility of the Daily Returns of Nigerian Insurance Stocks. International Business Research 13(2):106-116*.

Emenike, K.O. and Ani, W.U. (2014). *Volatility of the Banking Sector Stock Returns in Nigeria. Ruhuna Journal of Management and Finance, 1(1), 73-82.*

Engle, R.F.(1982). *Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. Inflation. Econometrica, 50, 987-1007.*

Hamilton, J. D. (2010). *Macroeconomics and ARCH in Tim Bollerslev, Jeffrey, R. Russel and Mark, W. Watson (Eds), volatility and time series Econometrics: Essays in Honor of Robert, F. Engle. Oxford University Press.*

Harvey, A. & Chakravarty, T. (2008). *Beta-t-(E) GARCH: Cambridge University Working Papers in Economics ACH34@ECON.CAM.AC.UK*.

Kalu O. E. (2010). *Modelling Stock Returns Volatility in Nigeria Using GARCH Models. Munich Personal RePEc Archive, No. 22723.*

Kalyanaraman, L. (2014). *Stock Market Volatility in Saudi Arabia: An Application of Univariate GARCH Model. Asian Social Science, 10(10): 142-152*

Kosapattarapim, C., Lin, Y. & McCrae, M. (2022). *Evaluating the volatility forecasting performance of best fitting GARCH models in emerging Asian stock markets. International Journal of Mathematics & Statistics, 22 (2), 1-15*

Liu, H.C. and J.C. Hung. (2010). *Forecasting S&P-100 stock index volatility: The role of volatility asymmetry and distributional assumption in GARCH models. Expert Systems with Applications, 37, 4928-4934*.

Mantalos,P.,Karagrigoriou, A.,Střelec,L.,Jordanova,P., Hermann, P., Kiseľák,J., Hudák,J and Stehlík, M. (2019). *On improved volatility modelling by fitting skewness in ARCH models.Pages 1031 1063, Published online: 30 Sep 2019*

Olowe, R. A. (2009). *Stock Return volatility and the global financial crisis in an emerging markets: the Nigerian case. International Review of Business Research papers, vol. 5,4: 426-447*

Suleiman, H. K. (2011). *Stock return and volatility persistence in Nigerian capital market. A P.hD. Post-data seminar submitted at Ahmadu Bello University, Zaria, Nigeria.*

Taylor, S. (1986).*Modelling Financial Time Series, Wiley, Chichester*.

Tsay, R. S. (2010)..*Analysis of Financial Time Series; 3rd Edition, Published by John Wiley &Sons, Inc., Hoboken, New Jersey. Published simultaneously in Canada*

Uyaebo, S. O., Atoi, V. N. and Usman, F. (2015). *Nigeria Stock Market Volatility in Comparison with some Countries: Application of Asymmetric GARCH Models. CBN Journal of Applied Statistics, 6(2): 133-160.*