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A Homotopical Framework for Modeling Continuous Deformations in Human Body Shape and Ageing

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Abstract

This study applies homotopy theory to model the continuous deformations in human body shape and ageing process. We extend the existing work by adding genetic factors and examining the deformation of the body and ageing process, where it is shown that for every growth process and ageing is attained, there exist a growth and ageing threshold for the deformation of the body. The homotopy concept is used to describe the corresponding connected surface of the structure of the body and ageing process. The ageing increases at time variable t. The performance of the continuous deformation functions $G_t(x)$, $H_t(x)$ and $F_t(x)$ on the map depend on the adjusted interval [0,1], [1,2], [2,3], [3,4], ..., [r,n] to produce the sequence of age up to deformation point at natural death which is the limiting point of the deformed body or point of discontinuity.

Keywords: Homotopy, Continuity, Connectedness, Ageing, Biological Deformation, Morphogenesis, Algebraic Topology.

Introduction

The study of topological spaces and continuous functions is central to the topology idea. The fundamental flaw is the methodical organisation to the degree that the functions and spaces are homomorphically arranged. The continuous deformation-based identical relation problem gives rise to yet another systematic arrangement challenge. In

algebraic topology, the latter is significant since it appears to have the most widely available and practical tools.

The study of topological spaces and continuous functions through algebraic objects like groups, rings, and homomorphism is known as algebraic topology. Topology and algebra are connected through mappings (Spinner, 1996). The algebraic invariants categorise topological spaces up to homomorphism (typically homotopy equivalence). The most significant of these invariants are homotopy groups, homology groups and co-homology groups (rings). Mohammed and Ghassan (2018) investigates Susceptible-Infected-Recovered (SIR) model with vital dynamics and constant population, which is used as a mathematical model in many physically significant fields of applied science. They used Homotopy Perturbation Method (HPM) and Runge-Kutta method (RK) for solving the SIR model with vital dynamics and constant population. The convergence of HPM has also been examined. The HPM was tested on solving different implementations and was shown to be efficient and accurate. The approximated solutions of HPM for the tested problems agreed with the numerical solutions of Runge-Kutta method.

The genetic properties, hormonal and other environmental factors are responsible for the growth of human body. Though, this is obvious, but therefore not stops researchers to make further study works. As the human body under goes changes with respect to time, ageing occurs. Ageing is shown in all parts of the body since the parts of the body are stuck to one another and have one piece with common surface. The common surface of the body makes it possible to study the behavior of the body. The ageing of the body is described by continuity and connectivity which are topological properties (Brew, et at., 2019).

The idea of continuity and connectivity has been used in many fields, for example in the vector calculus. In the vector calculus, it was used to proposed the relationship between a line integral around a simple closed curve and a double integral over the plane region bounded by the curve (Massey, 1991).

In the recent year, many Analytical Method and Numerical Algorithms have been utilized for solving the fractional diffusion equations such as the Homotopy Analysis Method (HAM) (Jafari et al., 2009), the Adomian Decomposition Method (ADM) (Daftardar-Gejji., 2010), the Homotopy Perturbation Method (HPM) (He and Nadeem, 2021) & (Jafari and Momni, 2007). Among these algorithms is the Homotopy Analysis Method (HAM) which was first introduced by the Chinese mathematician Liao in 1992 (Liao, 1992). The well known perturbation method or technique in the literature which is base on the existence of small or large parameter called the perturbation quantity was significantly improved by the idea of the Homotopy Analysis Method.

Computable invariants such as Euler characteristics and homology groups have been used to study the number of connected components of a space or object. Several researchers have used homology groups to study digital images. With the amount of work already carried out, it was established that the invariants provided the same information for all the closed and connected surfaces of the spaces or the objects (Boxer, 2011; Kinsey, 1993). Several attempts are still

under way to improve the invariants and develop the invariants, thereby making the area inactive area of research. A few of the developed invariants that reflect the connectivity properties of spaces or objects were derived from the topological chains but not much has been seen in the direction of the ageing process of the human body (Grandy and Polimeni, 2010).

Morphogenesis is described as a biological process where organisms develop their complex shapes and forms which includes coordinated cell activities such as division, migration and differentiation controlled by genetic signals to create tissues and organs from a single cell necessary for development and evolution. According to Zhiyi, et al. (2019), a morphogenesis is a process explaining how the shapes of living tissues and bodies are created during development.

Differential growth generates mechanical stresses that drive deformations at both cellular and tissue scales, shaping functional organ morphologies. This review introduces the core principles of growth mechanics in animal tissues and demonstrates how differential growth contributes to the generation of mechanical stresses that shape organs through processes such as folding, bending, and buckling, especially when different tissue layers or extracellular matrices impose external constraints (Erlich and Harmansa, 2025)

While homotopy theory has been widely applied in various fields, its potential in modeling human morphogenesis, particularly the continuous deformation of human body shape and ageing, remains underexplored. Despite extensive research on morphogenesis and ageing, a rigorous mathematical framework to describe biological shape deformation and ageing as continuous mappings is lacking. Existing studies have not fully leveraged homotopy theory to capture the complex, continuous changes in human body shape and ageing process, highlighting a significant gap in the literature.

The purpose of this research work is to demonstrate the application of the algebraic invariant to physical quantities up to homomorphism and to extend the homotopy notion to the algebraic topology for the body's deformation and ageing process. Additionally, to link homotopy to the ageing process of humans, to extend the idea of homotopy in algebraic topology, and to provide some pertinent topological advice for applying homotopy to other physical continuous processes.

Materials and Methods

Algebra of Limit and Continuous Function

Theorem 3.1. Giving L_1 , L_2 and a. Suppose that *f, g* and are real values function defined on some open interval I containing a, except possibly at a itself and that $\lim_{x\to a} f(x) = L_1$ and $\lim_{x\to a} g(x) = L_2$ then;

i. $\lim_{x\to a}[f(x) \pm g(x) = L_1 \pm L_2$
ii. $\lim_{x\to a}[f(x).g(x) = L_1 . L_2$
iii. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L_1}{L_2}$; provided $g(x) \neq 0\ \forall x \in I$ and $L_2 \neq 0$

Proof

(i) Let $\epsilon \geq 0$ be given, then there exist $\{\delta_1 > 0\}$ and $\{\delta_2 > 0\}$ such that $|f(x) - L_1| < \frac{\epsilon}{2}$ whenever $x \in I$ and $0 < |x - a| < \delta_1$ and $|g(x) - L_1| < \frac{\epsilon}{2}$ whenever $x \in I$ and $0 < |x - a| < \delta_2$. Let $\delta = max\{\delta_1, \delta_2\}$. Then whenever $x \in I$ and $0 < |x - a| < \delta$

We have:

$$|[f(x) + g(x)] - [L_1 + L_2]| = |[f(x) - L_1 + g(x) - L_2]||[f(x) + g(x)] - [L_1 + L_2]|$$
$$< |f(x) - L_1| + |g(x) - L_2||[f(x) + g(x)] - [L_1 + L_2]|$$

$$|[f(x) + g(x)] - [L_1 + L_2]| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

Thus,

$$\lim_{x \to a}[f(x) + g(x) = L_1 + L_2$$

A similar argument shows that

$$\lim_{x \to a}[f(x) - g(x) = L_1 - L_2$$

(ii) With $\epsilon > 1$, there exist $\delta_1 > 0$ such that $|f(x) - L_1| < 1$, whenever $x \in I$ and $0 < |x - a| < \delta_1$

This implies that

$|f(x)| < |f(x) - L_1| + |L_1| < 1 < 1 + L_1$. Whenever $x \in I$ and $0 < |x - a| < \delta_1$

Now we have,

$$|[f(x)g(x)] - [L_1L_2]| = |f(x)g(x) - f(x)L_2 + f(x)L_2 - L_1L_2|$$

Then

$|[f(x)g(x)] - [L_1L_2]| \leq |f(x)||g(x) - L_2| + |L_2||f(x) - L_1|$ given that $\epsilon > 0$ there exist $\delta_2 > 0$ and $\delta_3 > 0$

such that

$$|f(x) - L_1| < \frac{\epsilon}{2(1 + L_2)}$$

whenever $x \in I$ and $0 < |x - a| < \delta_1$ and let $\delta = max\{\delta_1, \delta_2, \delta_3\}$. Then, whenever $x \in I$ and $0 < |x - a| < \delta$

we have,

$$|[f(x)g(x)] - [L_1L_2]| \leq \left|(1 + L_2)\left[\frac{\epsilon}{2(1 + L_2)}\right] + L_2\left[\frac{\epsilon}{2(1 + L_2)}\right]\right|$$

$$< \left|(1+L_2)\left[\frac{\epsilon}{2(1+L_2)}\right] + (L_2+1)\left[\frac{\epsilon}{2(1+L_2)}\right]\right| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

Thus:

$$\lim_{x\to a}[f(x)g(x)] = L_1L_2$$

(iii) It is enough to show that $\lim_{x\to a}\frac{1}{2} = \frac{1}{L_2}$ provided that $g(x) \neq 0\ \forall x \in I$ and $L_1 \neq 0$. $Since\ L_2 \neq 0, \epsilon = \frac{L_2}{2} > 0$. Therefore, there exist a $\delta > 0$

Such that:

$|g(x) - L_2| < \frac{|L_2|}{2}$

Whenever $x \in I$ and $0 < |x-a| < \delta$. Now $\forall x \in I$ satisfying $0 < |x-a| < \delta$,

We have:

$|L_2| \leq |L_2 - g(x)| + |g(x)| < \frac{|L_2|}{2} + |g(x)|$

which implies that $|L_2| - \frac{|L_2|}{2} < |g(x)|$

that is $\frac{|L_2|}{2} < |g(x)| \forall x \in I$ satisfying $0 < |x-a| < \delta_1$. It now follow that all $x \in I$ satisfying $0 < |x-a| < \delta_1$. .

We have:

$\left|\frac{1}{g(x)} - \frac{1}{L_2}\right| \leq \frac{2|L_2 - g(x)|}{L_2^2} < \epsilon\frac{L_2^2}{2}.\frac{2}{L_2} = \epsilon$

Thus:

$\left[\lim_{x\to a}\frac{2}{L_2}\right] = \frac{1}{L_2}$

And from part (b) of the theorem we have:

$\left[\lim_{x\to a}\frac{f(x)}{g(x)}\right] = \left[\lim_{x\to a}\frac{1}{g(x)}f(x)\right]$

$\left[\lim_{x\to a}\frac{f(x)}{g(x)}\right] = \frac{1.L_1}{L_2}\left[\lim_{x\to a}\frac{f(x)}{g(x)}\right] = \frac{L_1}{L_2}$

Theorem 3.2

Let X $\subseteq R^n$ and Y $\subseteq R^m$. A function $f: X \to Y$ is continuous at a point $x \in X$ if and only if $\in > 0$ there exist a $\delta > 0$, such that whenever $a \in X$ and $\|x-a\| < \delta$, then $\|f(x) - f(a)\| < \in$.

Proporsition 3.1

If f is continuous function on close interval $[a, b]$ and $f(a) \neq f(a)$, then for each number K between $f(a)\ and\ f(a)$, there is a point C$[a, b]$ such that $f(x)$ = K.

Proof

For definiteness, assume that $f(a) = f(b)$, Let S = $x \in [a, b]: f \leq K$. Then $S = \emptyset$ since $a \in S$, then $C = sup\ S$ exist as a real number in $[a, b]$, there exist a sequence (x_n) in S such that $a \leq c \leq b$ and so f is continuous at C. This implies that:

$\lim_{x \to a} f(x_n = f(c))$

As $f(x_n) \leq k$ for each $n \in N$, we deduce that $f(c) = k$, and also $c \in S$. It remains to show that since $c \in S$ and $c = supS$, $c + \frac{1}{n} \in S$ for each $n \in N$. Also, since $K \in f(b)$, we have that $c < \frac{1}{n} < b - cS$ that is $c < \frac{1}{n} \in [a, b]$ and $c + \frac{1}{n}$. Thus $f(c + \frac{1}{n}) > k$ for all $n \geq N$. By the continuity of f we obtain that $f(c) \geq k$, hence $f(c) = k$.

Theorem 3.3 (Continuity Theorem)

Let D $\subset \Re$ and $f: D \to \Re$. Then f is continuous at $a \in D$. if and only if fir every sequence $(a_n) \subset D$ such that $\left[\lim_{x \to a} a_n = a\right]$ we have that $\left[\lim_{x \to a} f\ (a_n) = f(a)\right]$.

Proof

Suppose that f is continuous at $a \in$ D and that (a_n) a sequence in D such that the $\left[\lim_{x \to a} a_n = a\right]$. Given that $\in > 0$ there exist $\delta > 0$ and an $N \in N$ such that that $| f(x) = f(a) | < \in \ \forall\ n \in N$. Whenever $| x - a | < \delta$ and $| x - a_n | < \delta$, for all $n \geq N$.

Therefore $| f(x_n) - f(a_n)| < \in \ \forall\ x \in N$. That is $f(a_n) = f(a)$.

For the inverse, we assume that for every sequence $(a_n) \subset$ D such that $\left[\lim_{x \to a} a_n = a\right]$, we have $\left[\lim_{x \to a} f(a_n) = f(a)\right]$ and that f is not continuous at a. Then there exist $\epsilon_0 \geq 0$ such that for every $\delta \geq 0$, with $0 < |x - a| < \delta$ we have $|f(x) - f(a)| \geq \ \epsilon_0\ \forall\ n \in N$, let $\delta = \frac{1}{n}$ and then we can find $a \in D$ such that $0 < |a_n - a| > \frac{1}{n}$ and $|f(a_n) - f(a)| \geq \ \epsilon_0$.

Clearly (a_n) is a sequence in D with properties that $\left[\lim_{x \to a} f(a_n) = f(a)\right]$ and $[f(a_n) = f(a)] \geq \epsilon_0\ \forall\ n \geq N$. That is continuous.

Homotopy

Lamma 3.1 (Homotopy Equivalent)

If f and g are homotopic, then $H_n(g, D) = H_n(g, D)$ is the homotopy invariance. It follows that of x and Y are homotopy equivalent, then $H_x(g, X)$ is isomorphic to $H_n(Y, D)$ [12].

Lamma 3.2 (Homotopy Theorem)

The operator $(*)$ has the following properties:

(i) Associativity: If $[f] * ([g] * [h])$ is defined, so is $[f] * [g] * [h]$, and they are equal.

(ii) Right and left identity: Given $x \in X$, Let e_x denote the constant path: e_x: I $\rightarrow X$ carry all of 1 to the point x. If f is a path in x from x_0 to x_1, then: $[f] * [e_x] = [f]$ and $[e_x] = [f]$.

(iii) (Inverse). Given the path f in x from x_0 to x_1. Let f be the path define by $f(s) = f(1-s)$. It is called the reverse of f. Then $f[f][f'] = f[f'][x_1]$.

Connectedness

Proporsition 3.2

Suppose $X \subseteq \Re$ is connected if and only if it's on interval.

Proof

Case1: Suppose that $[a, b]$ is not connected. Let $[a, b] = v_1 \cup v_2$ (such that the interval is disjoint), hence v_1 is open, since I = 1,2,3…. Without lost of generality (wlgo) we need to show that v_1 not empty as we let $a = v_1$ and sufficient we show v_2 is empty that is $v_2 = \phi$. Let $J = \{X|[0, x] \subseteq v_1\}$, let $C = lubJ > a$. We use some properties of real space, that is C' with $[a, x] \subseteq v_1$ such that $[a, c'] \subseteq v_1$and $[a, c] \subseteq v_1$.

Case2: We shall consider case two in the following

(i). Suppose that $C \in v_2$ which implies that $(c-\varepsilon, c+\varepsilon) \subseteq v_2$ for some $v_2 > 0$ then $v_2 \cap v_1 = \phi$.

(ii). Let $C \in v_1 and\ v_1$ is open, it implies that $c-\varepsilon, c+\varepsilon) \cap [a, b] \subseteq v_1$. also $(a, c+\varepsilon) \cap [a, b] \subseteq v_2$. Hence $c = lubJ$ which implies that $c = b$.

We want to show that v_2 is empty, $v_2 = v_1^c \subseteq [a, b]^c = [a, b]$. Thus $v_2 = \phi$ and $[a, b]$ is connected.

Lamma 3.3

X $\subseteq \Re$ is connected if and only if it's an internal.

Corrolary 3.1

Let $f: X \rightarrow Y$ be connected and X which is the domain be connected, then $f(x)$ which is the image with subspace topology is connected.

Proporsition 3.3 X is locally path connected if and only if path components are open.

Results

This research work relies on the properties of topological space. We consider the human body (x) as the topological space since our aim is to apply homotopy concept into the algebraic

topology for the bological deformation of the body where growth stops and at natural death without the influence of environmental factor such as diseases, harzadous event and accident to the deformation of the body growth and ageing process. We intent to construct an invariants such as homotopy up to homomorphism that reflect the connectedness of the body and it developmental stages.

We consider the human body $(x) = x$ with the three state major variables homotopies: $F(x)$ denoting the ageing process at time (t), $G(x)$ denoting genetic properties and $H(x)$ denoting the growth developmental stages of the body. We define a mapping body $G: H \to F$ since the genetic functions induce the developmental growth of the body and the developmental functions induce the ageing process. Hence, we develop a theorem for the fundamental homomorphism as shown below.

Theorem 4.1

If $G: H \to F$ is a homomorphism of a group H into another group F, then ker (G) is a normal subgroup of H and the mapping $\psi: G(H) \to H/N$ define by $\psi: (G(H)) = N_h$ where $N = \ker(G)$

Proof:

Case1:

We want to prove five (5) things such that if it is (i) subgroup (ii) normal (iii) homomorphism (iv) mapping (v) isomorphism.

We show that:

$$N = \ker(G)$$

Such that $N = \phi$ is a subgroup of H. We observe that $\neq \phi$, since;

$$G(e) = e'$$

and thus $e \in N$. Now let $n_1, n_2 \in N$, then;

$$G(n_2) = e'$$

If we pick any element inside the kernels, then the image must be an identity. Now;

$$G(n_1 n_2^{-2}) = G(n_1)G(n_2^{-2})$$
$$G(n_1 n_2^{-2}) = G(n_1)G[(n_2^{-2})]$$
$$G(n_1 n_2^{-2}) = e'(e')^{-1}$$

Thus;

$$G(n_1 n_2^{-2}) = e'$$

Hence, N is subgroup of H

Case 2:

Now we show that N is normal in H. Let $n \in N, h \in H$ but we know that $G(n_1) = e'$, where $i = 1,2$ and by definition

$$G(hnh^{-1}) = G(h)G(n)G(h^{-1})$$
$$G(hnh^{-1}) = G(h)e'G[(h)]^{-1}$$

$$G(hnh^{-1}) = G(h)e'[e]^{-1}$$

Thus

$$G(hnh^{-1}) = e'$$

it implies;

$$G(hnh^{-1}) = e' \; \forall \; hnh^{-1} \in N. Thus \; N \; is \; normal \; in \; G$$

Case 3:

Consider $\rho: G(H) \rightarrow H/N$ define by;

$$\rho\big(G(H)\big) = N_h$$

We show that ρ is a mapping, it is possible that $h_1 \neq h_2$

Thus, it implies

$$G(h_1) = G(h_2) \; \forall \; h_1, h_2 \in H$$

Now

$$G(h_1 h_2^{-1}) = G(h_1)G(h_2^{-1})$$

$$G(h_1 h_2^{-1}) = G(h_1)G(h_2^{-1})$$
$$G(h_1 h_2^{-1}) = G(h_1)G[(h_2^{-1})]$$
$$G(h_1 h_2^{-1}) = G(h_1)e'$$

Hence;

$$G(h_1 h_2^{-1}) = e'$$

It implies that $h_1, h_2 \in N, h_1 \in N_{h2}$ and similarly we can have $h_1 \in N_{h2}$ and it is the cosets form. We have that

$$N_{h1} = N_{h2}$$

which implies that;

$$\rho\big(G(h_1)\big) = \rho G\big((h_2)\big)$$

Case 4:

Now, we prove that ρ is a homomorphism. Consider the transformation:

$$\rho[G(h_1)G(h_2)] = \rho[G(h_1 h_2)]$$
$$\rho[G(h_1)G(h_2)] = N_{h1}h_2$$
$$\rho[G(h_1)G(h_2)] = N_{h1}.N_{h2}$$

Thus:

$$\rho[G(h_1)G(h_2)] = \rho G(h_1).\rho G(h_2)$$

Hence it is a homomorphism.

Case 5:

Finally, we show that it is an isomorphism, (i.e one-to-one or onto). ρ is one-to-one for if:

$$\rho\big(G(h_1)\big) = \rho G((h_2))$$
$$N_{h1} = N_{h2}$$

and,

$$G(h_1) = G(nh_2)$$
$$G(h_1) = G(n)(h_2)$$

Finally:

$$G(h_1) = G(h_2)$$

Therefore, it is onto and since it is it is classified up to homotopy equivalent and hence the prove.

Theorem 4.2

Let $N \rhd H$ and G be a subgroup of H. Then; (i) if FnN is subjective to F, we need to prove it is normal in F; (ii) we need to prove that $FN \neq \phi$; (iii) $G: F \to FN/N$.

Proof:

Case1:

Let $f \in F, \alpha \in (FnN)$, it implies that $f^{-1}\, \alpha\, f \in (FnN)$. Thus $FnN \rhd F$.

Case2:

Obviously $FN = \phi$. Now let $\alpha_{1,}\alpha_2 \in FN\ \exists\ \alpha_1 = f_1 n_1$ and $\alpha_2 = f_2 n_2$ where $f_1 f_2 \in F$ and $n_1 n_2 \in N$.

Consider:

$$\alpha_{1,}\alpha_2 = (f_1 n_1\ ).\ (f_2 n_2)$$
$$\alpha_{1,}\alpha_2 = \ (f_1 n_1\ ).(f_2^{-1} n_2^{-1}\ )$$
$$\alpha_{1,}\alpha_2 = \ [f_1 (n_1 n_2^{-1}) f_2^{-1}]$$

Therefore;

$$\alpha_{1,}\alpha_2 = \ [f_1 (n_3^{-1}) f_2^{-1}$$

Where

$$n_3 = n_1 n_{2.}$$

We introduce

$$h_1 h_2^{-1}.$$

Thus;

$$f_1 (f_2 f_2^{-1}) n_3 f_2^{-1} = (f_1 f_2^{-1}) f_2 n_3 h_2^{-1}$$
$$f_1 (f_2 f_2^{-1}) n_3 f_2^{-1} = (f_1 f_2^{-1}) n_3 f_2 h_2^{-1}$$
$$(f_1 f_2^{-1}) n_3 f_2 h_2^{-1} = f_n$$

Where

$$f_1 f_2^{-1} = n_3 f_2 h_2^{-1}$$
$$f_1 f_2^{-1} = n\ \forall\ n\ \in FN.N$$

Being normal. Hence HN is a subgroup of F.

Case3:

By defining a mapping $G: F \to FN/N$ define by:

$$G(F) = NF$$

Also G is a homomorphism;

Thus;

$$G(f_1 f_2) = NG(f_1 f_2)$$
$$G(f_1 f_2) = Nf_1 . Nf_2$$

Therefore;

$$G(f_1 f_2) = G(f_1).G(f_2)$$

By property of homomorphism.

Again G is onto for if $Nf \in FN/N \; \exists \; f \in F$ such that:

$$G(f) = Nf$$

Thus;

$G: F \to FN/N$ is a homomorphism by applying the first law of homomorphism theorem.

Then;

$$G(F) \cong F/\ker\,(G).G(F)$$
$$G(F) \cong N/N$$

Thus;

$$FN/N \cong F/\ker\,(G)$$

Which implies that;

$$\mathrm{Ker}(G) = \{f \; \in F/F(f)\}$$
$$= \{indentity\ image\}$$
$$\mathrm{Ker}(G) = \{f \in F/f \in F\}$$

So that;

$$\mathrm{Ker}(G) = FnN$$

Therefore;

$$\mathrm{F/FN} \cong FN/N$$

To means that F/FN is homomorphic to FN /N and so proved of the algebraic invariant that reflect the connectedness of the body.

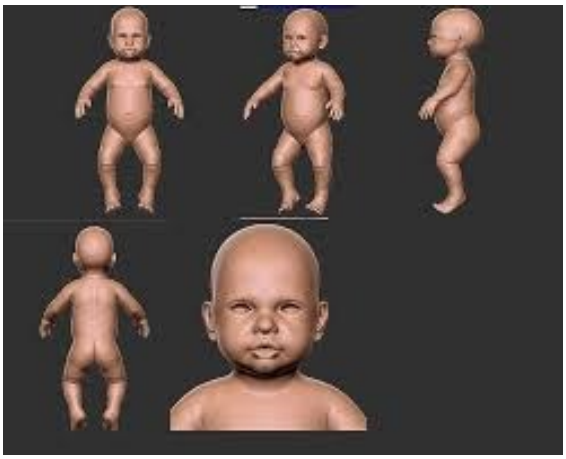
Homotopy of the Ageing Process

We consider the values of time parameter (t) and the topological body (x) in the function $f_t\,(x)$ which when adjusted to control the process of ageing of the body. If for the interval $t \in [0,1]$, there exist a homotopy $f_t\,(x)$ from $f(x)\ to\ h(x) * g(x)$ such that $f_0(x) = f(x) * g(x)$, and $f_1(x) = h(x) * g(x)$ then the body is only one year old. Similarly, if for the interval $t \in [1,2]$, there exist a homotopy $f_t\,(x)$ from $f(x)\ to\ h(x) * g(x)$ such that $f_1(x) = f(x) * g(x)$, and $f_2 = h(x) * g(x)$ then the body is two years old. If for interval interval $t \in [2,3]$, there exist a homotopy $f_t\,(x)$ from $f(x)\ to\ h(x) * g(x)$ such that $f_2(x) = f(x) * g(x)$, and $f_3(x) = h(x) * g(x)$ and this continuous.

Thus for the interval $t \in [r, n]$, where $n > a$, there is continuous function $f_t\,(x)$ called the homotopy from one function $f(x)$ to another function $h(x)$, and $g(x)$ satisfying the condition $f_r(x) = h(x) * g(x)$ and $f_n(x) = h(x) * g(x)$ then the body is described to be n years old,

where $n = 1,2,3 \ldots \ldots N$ and $N = \infty$, the age threshold. Hence both the age and the topological age of the body grow as the homotopy increase from $f_r(x) = h(x) * g(x)$ to $f_n(x) = h(x) * g(x)$. For interval $t \in [r, n]$. It is observed that the interval $t \in [r, n]$ is the supremum $f_n(x) = h(x) * g(x)$ is the actual age of the body. The age increase along 1-dimension. In homology it is 1-simplex or a line or a path. But the body (x) is a closed connected space or surface such that at each change in time variable, the age of the body (x) increase from one level to another. At adulthood where the body no longer enlarge or grow and at death, the ageing and the growth of the body deformed, so it continuous not.

Visual Model of the Body for Ageing Process



a) Infant



b) Teenager



c) Adult

Fig. 1. *Visual model of the body for ageing process from infant to teenager to adult*

Homotopy of the Growth Process

We showed the mechanism by which morphogenesis play a vital role in the biological process that causes a cell tissue or organism to develop or grow it shape. It takes embrayological

process of differentiation of cell, tissue, and organs and the development or growth of organ systems to acquire dimension and form. We consider the values of the time parameter (t) and (x) in the function $h_t(x)$ which when adjusted to control the process of growth of the body. If for interval $t \in [0,1]$, there exist a homotopy $h_t(x)$ from $h(x)$ to $f(x) * g(x)$ such that $h_0(x) = f(x) * \mathrm{g(x)}$, and $h_1(x) = h(x) * \mathrm{g(x)}$ then the body is only one year old. Similarly, if for interval $t \in [1,2]$ there exist a homotopy $h_t(x)$ from $h(x)$ to $f(x) * g(x)$ such that $h_1(x) = f(x) * g(x)$ and $h_2(x) = f(x) * g(x)$, then the body is two year old. If for interval $t \in [2,3]$ there exist a homotopy $h_t(x)$ from $h(x)$ to $f(x) * g(x)$ such that $h_t(x) = f(x) * g(x)$ and $f_2(x) = h(x) * g(x)$ and $f_3(x) = h(x) * g(x)$, then the body is three years old. This continuous from infant, teenage, and adulthood.

Thus for the interval $t \in [r, n]$ where $n > a$, there is a continuous function $f_r(x)$ called the homotopy from one function $f(x)$ to others functions $h(x)$ and $g(x)$ satisfying the condition $f_r(x) = h(x) * g(x)$ and $f_n(x) = h(x) * g(x)$ then the body is described to be n years old, where $n = 1,2,3 \ldots \ldots . N$ and $N = \infty$, the age threshold. Hence both the age and the topological age of the body grow as the homotopy $f_t(x)$ increase from $f_r(x) = h(x) * g(x)$ to $f_n(x) = h(x) * g(x)$ is the actual age of the body. The age increase along 1-dimension. In homology it is 1-simplex or a line or a path. But the body (x) is a closed connected space or surface such that at each change in time variable, the age of the body (x) increase from one level to another. At adulthood where the body no longer grow but enlarge due to diet intake or at death where the ageing and the growth of the body come to deformed state, so it continuous not.

Visual Model of the Body for Growth Process

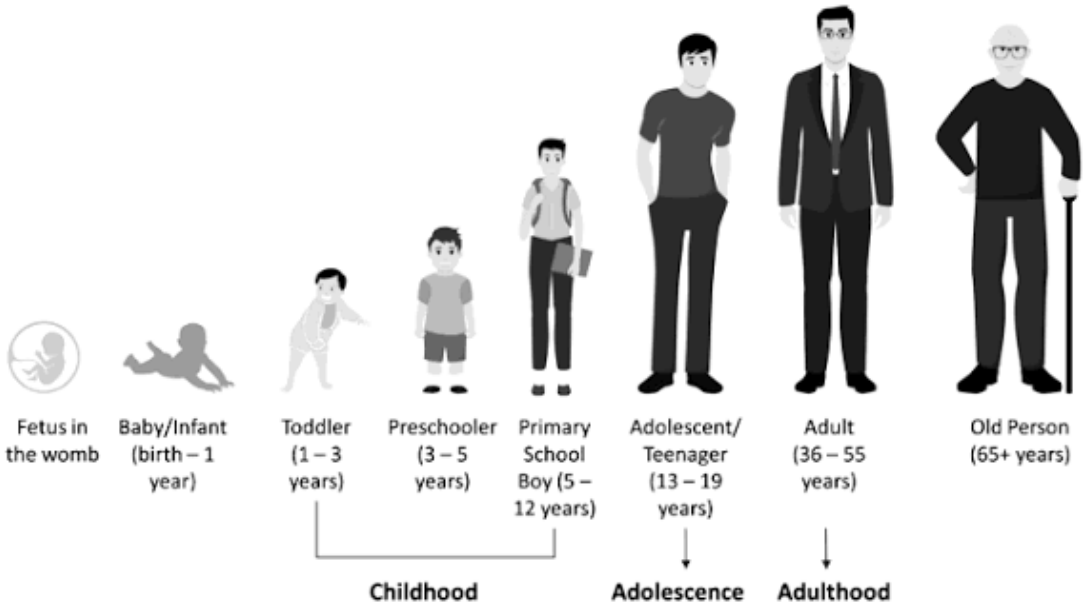


Fig. 2. *Visual model of the body for growth process*

Homotopy of the Deformation Point

For the deformation point, if for the interval $t \in [r, n]$ where $n > a$, there is a continuous function $f_t(x)$ called the homotopy from one function $f(x)$ to others function $h(x)$, $g(x)$ satisfying the condition $f_t(x) = h(x) * g(x)$ and $f_n(x) = h(x) * g(x)$ then the body is described to be n year old, where $n = 1,2,3 \ldots . N$ and $N = \infty$, the age threshold. Hence both the age and the topological age of the body grow as the homotopy $f_r(x)$ increase from $f_r(x) = h(x) * g(x)$ and $f_n(x) = h(x) * g(x)$ for the interval $t \in [r, n]$. It is observed that the interval $t \in [r, n]$ is the supremum $f_n(x) = h(x) * g(x)$ is the actual age of the body.

Visual Model of the Body for Deformed Point

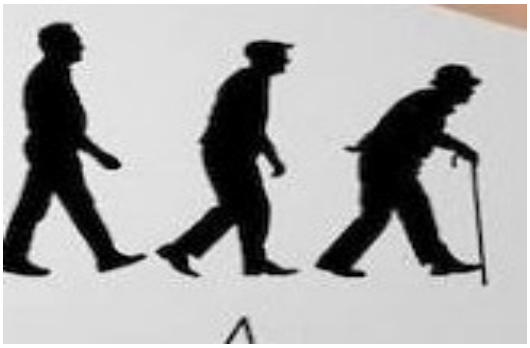


Fig. 3. *Visual model of the deformed body at old age*

Discussion

In this research work, we consider the human body $(x) = x$ as a topological connected space $[\alpha, \beta]$. We also showed the mechanism by which the genetic property $g(x)$ of the zygote may determine the anatomical structure of the body in terms of the growth and ageing process of the body which manifest it self in resemblance, color, height, stature or shape etc.

We found that ageing property $f(x)$ and the developmental or growth property $h(x)$ deformed at threshold. It has been evident in the construction of the algebraic invariance that reflect the connectedness of the body that growth in size, height, and age thus led to deformation at age and growth thresh holds, this was achieve by extending the work of authors in (Brew et, al., 2019).

The authors were able to show the topological shape of the wrinkle body using a cylindrical object from $f(x)$ to $g(x)$ and ageing process on the interval $I = [0, \beta]$ and the enlargement or growth and ageing of the body on the interval $I = [\beta + \prod]$ years from $f(x)$ to $g(x)$ showing the new age and growth of the body (x). Adding to their work we extent it by considering the biological factors such as hormonal and genetic properties, we construct an invariant of the human body up to homomorphism that reflect the connectivity of the body using the fundamental theorem of homomorphism, we also developed theorem and proved them in this regard. After which we present the topological shape of the human body showing the ageing and growth process, we have found that the map $f_t(x)$ is increasing by one simplex

or dimension as we generate the sequence of ageing and growth process and found that when adjusted in the interval $[0,1], [1,2], [2,3], [3,4], \ldots\ldots, [0,\beta]$ respectively, we conclude that the body grow at 1-simplex or dimension as the age also increase up to the age and growth end point known as the deformation point.

Conclusion

In this work, the researchers extend an existing work by adding genetics and examined the deformation of the body and ageing process, where it is showed that for every growth process and ageing is attained, there exist a growth and ageing threshold for the deformation of the body.

The study also proposed an approach on the application of the algebraic invariant to physical quantities or phenomenon. The proposed approach was successful in establishing the growth in the age of the body and the deformation at natural death. A Homotopy concept was used to describe the corresponding connected surface of the structure of the body and ageing process. The ageing increases at time variable t. The performance of the continuous deformation functions $G_t(x), H_t(x)\ and\ F_t(x)$ on the map depend on the adjusted interval $[0,1]$ $[1,2]$ $[2,3]$ $[3,4]$ $[r,n]$ to produced the sequence of age up to deformation point at natural death which is the limiting point of the deformed body or point of discontinuity.

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